INVESTIGACIONES ECONOMICAS. Volumen XIX (1) - Enero 1995, pp. 3-34

INCOME DISTRIBUTION AND SOCIAL WELFARE: A REVIEW ESSAY

Javier RUIZ-CASTILLO*
Universidad Carlos III de Madrid

Taking advantage of some of the lessons learned from income inequality comparisons over time and/or across space, we provide a complete framework of analysis to compare the social or aggregate welfare of independent cross-sections of household income and non-income household characteristics. This framework serves to clarify a number of traditional issues on I) the proper domain of the social evaluation problem; II) the need to consider alternative mean invariant inequality notions; III) the decomposition of changes in real welfare into changes of the mean at constant prices and changes in real inequality; IV) the nature of the inter-household welfare comparability assumptions implicit in all empirical work, and V) the strong implications of separability assumptions necessary for inequality and welfare decomposition by population sub-groups.

1. Introduction

In every science, there is no interesting measurement without theory. In addition, in the social sciences there is no measurement without value judgements. These lessons are well established in many parts of economics. In particular, since the seminal work of Atkinson (1970), Kolm (1976a, b), and Sen (1973), following up on the classical paper by Dalton (1926), the empirical analysis of income distributions belongs to the field of welfare or normative economics.

The value of any empirical piece in this area depends on the data available, and on the choice of a conceptual framework within which the measurement exercise becomes meaningful. Here, we provide a complete framework of analysis to compare the social or aggregate welfare of independent cross-sections of household data on total income, expenditures in particular commodities, and non-income characteristics. In so doing, we will be taking advantage of some of the lessons learned from income inequality comparisons over time and/or across space.

* This work is part of a larger project carried on under the auspices of the chair *Gumersindo de Azcárate*, presently occupied by the author and financed by the *Ministerio de Asuntos Sociales* of Spain. Support from Project PB89-0103 of the Spanish DGICYT is also acknowledged. Following many of the indications offered by two referees, the final version was much improved. All remaining errors and shortcomings are the author's responsability.

In the individualistic and welfarist tradition to which this paper belongs, one starts modelling individual agents and then proceeds to examine aggregation procedures. Ideally, for welfare purposes we need two items at the individual level: a set of unconditional household preferences, defined on commodities and household characteristics, and a way to go from household welfare to the welfare of the persons making up each household. For the aggregation part, we need a social evaluation function (SEF from here on) embodying all the relevant value judgements from an ethical point of view. The fundamental question is which should be the class of admissible SEFs for empirical analysis.

To clarify the issues involved, we first consider the simplest possible case: a homogeneous population of identical individuals who are only allowed to differ in a single dimension called income. Although, in principle, the SEF should be defined in utility space, making use of the indirect utility function we can examine the relationship between utility levels, incomes, and prices. The *named goods* approach, where we care about the allocation of commodities to households, is integrated in this framework. Our discussion of the SEF's domain concludes with a remainder of the fundamental difficulties for justifying welfare analysis in income space on the belief that, under general conditions, its conclusions will carry over onto utility space.

In income distribution theory, certain conditions are generally assumed without much questioning: symmetry, a preference for equality, and continuity. However, all known empirical procedures impose other restrictions on an admissible SEF which are based –more or less informally– on two closely related ideas. On the one hand, in the ethical approach to the measurement of inequality one would like to use a SEF to which one can associate, in a consistent way, only one inequality index. On the other hand, it is very convenient to work with a SEF which can be expressed in terms of only two statistics of the income distribution: the mean, and a measure of inequality.

As Dutta and Esteban (1992) have shown, to achieve these objectives we need to specify the type of mean-invariance property we want our inequality indices to satisfy. This is politically important, since we know from the early discussion in Kolm (1976a) that the choice of a mean-invariance class of inequality measures is not merely a technical matter, but a value ladden question. Moreover, recent reports based on questionnaires indicate that people are by no means unanimous in their choice between relative, absolute or other intermediate notions of inequality [1].

Here, we will consider only the two polar cases of scale-invariant and translation-invariant inequality measures, or indices of relative and absolute inequality, respectively. This amounts to the requirement that the SEF satisfies what Dutta and Esteban call weak homotheticity in the relative case,

[1] See, for instance, Amiel and Cowell (1992), Harrison and Seidl (1991), and Ballano and Ruiz-Castillo (1993).

and weak translability in the absolute case. To ensure that social welfare responds positively to increases in the mean, the SEF must satisfy also an appropiate monotonicity condition in either case. To permit comparisons of populations of different size, a condition on social welfare invariance under population replications is usually assumed. This completes a minimal axiom set characterising what we call the standard model for welfare analysis.

The standard model captures a preference for efficiency and a preference for equality, but is silent on the trade offs between these two desirable aims. If one wants a complete welfare ordering of income distributions, further restrictions on the class of SEFs are needed. Here we will restrict most of our attention to the multiplicative and the additive trade offs between efficiency and equity which result from the admission, respectively, of homothetic and translatable SEFs. Of course, completeness and specific trade-offs are bought at a cost: going beyond the standard model, means accepting greater degrees of household welfare comparability in our measurement procedures.

Finally, we must confront the fact that populations are heterogeneous. Most societies can be partitioned along many dimensions which are important to distinguish for political, social or economic reasons. The novelty, perhaps, is that for an increasing number of countries we have very rich microeconomic information on both household income or expenditures, as well as geographic, demographic, educational and other socioeconomic characteristics. Because some of these dimensions –typically the demographic ones– entail different needs, economists and others have been worrying extensively about them from a normative point of view.

Three issues related to the heterogeneity of the population will be discussed. In the first place, extensions to the non-homogeneous case of stochastic dominance criteria, which provide only a partial ordering of all income distributions, are contrasted with the typical use of a single set of unconditional preferences to adjust households' income for price changes and non-income needs.

In the second place, if we accept the convenience of having SEFs decomposable by population subgroup, then we believe that aggregate welfare should be something more than the weighted sum of welfare within each of the subgroups. In particular, it seems intuitively acceptable that if the inequality between the subgroups –however defined– increases, aggregate welfare should decrease. It is seen that, together with the rest of the axioms reviewed, welfare decomposition into a within-group term and a between-group inequality component, leads, in the relative case, to a SEF which involves the distribution mean and Theil's first index of inequality and, in the absolute case, to the Kolm-Pollak family of SEFs.

In the third place, we admit that the assumption of a common unconditional utility function, necessary for the pooling of all units into a single distribution of comparable or equivalent incomes, only permits inter-household, not inter-personal, welfare comparisons. In the abscence of information on how total income is shared by persons inside the household, we can still discuss extensions of the domain of the social evaluation problem by means of different schemes for weighting the adjusted income distribution by different measures of household size. We suggest how Daltonian transfers should be defined in adjusted income space under alternative weighting schemes to avoid certain paradoxes raised by Glewwe (1991).

It should be said at the outset that there is little new in this paper. It is rather a review essay with an operational aim in mind, which extends and updates the treatment found, for example, in Deaton and Muellbauer (1980). Nevertheless, we believe that the suggested framework serves to clarify a number of traditional issues on I) the proper domain of the social evaluation problem; II) the need to consider alternative mean invariant inequality notions; III) the decomposition of changes in real welfare into changes of the mean at constant prices and changes in real inequality; IV) the nature of the inter-household welfare comparability assumptions implicit in all empirical work, and V) the strong implications of separability assumptions necessary for inequality and welfare decomposition by population subgroups. Perhaps, the main novelty is the realization of the great simplification achieved if income adjustment procedures for taking into account non-income needs are independent of household utility levels –an assumption originally introduced in the theoretical literature by Lewbel (1989) and Blackorby and Donaldson (1993) in the relative case, which was extended in Blackorby and Donaldson (1994) to the absolute case. If the paper conveys a sense of the benefits of the axiomatic method for welfare measurement, and contributes to justify or improve current empirical procedures, it will have satisfied its purpose.

The rest of the paper consists of three sections. In Section 2, we present the standard model for a homogeneous population, other assumptions on the trade off between equity and efficiency, and the distributional role of price changes. Section 2, which is devoted to the non-homogeneous case, contains a comparison of stochastic dominance criteria for bivariate distributions *versus* complete approaches leading to strong cardinal conclusions, a presentation of additively decomposable welfare measures, a discussion of the simplifying implications of making income adjustment procedures independent of household utility levels, and a review of alternative ways of weighting households for social evaluation. Section 4 concludes with a summary of assumptions and functional forms.

2. The homogeneous case

2.1. The domain of the social evaluation function in the simplest possible case

Assume we have a population of H identical households facing a price vector p in $\mathbb{R}^L_+$. They are characterised by their income x^h, and a utility function U^h, defined on the commodity space $\mathbb{R}^L_+$. Since all households are identical in every non-income dimension, there is little harm in assuming that there exists a common utility function for all of them, ruling out the case of identical individuals being different pleasure machines. Thus, we assume:

$$U^h(q^h) = U(q^h) \text{ for all } h = 1, \ldots, H,$$

where q^h is a commodity vector in $\mathbb{R}^L_+$.

Under general conditions on the direct utility function, there will exist an indirect utility function φ and an expenditure or cost function c. Thus, in a given sample of utility maximising and price taking consuming units, the observable data on prices, incomes, and commodity demands for each h are related by

$$u^h = U(q^h) = \varphi(x^h, p)$$

and

$$x^h = c(u^h, p).$$

A Social Evaluation Function (SEF) is a real valued function S defined in the space $\mathbb{R}^H$ of utility vectors, with the interpretation that for each utility vector $u = (u^1,\ldots, u^H)$, $S(u)$ provides the «social» or, simply. the aggregate welfare from a normative point of wiew[2]. Using the indirect utility function common to all units, the connection between utilities, incomes, and prices will be:

$$S(u) = S[\varphi(x^1, p), \ldots, \varphi(x^H, p)] = F(x^1, \ldots, x^H, p) = W(x^1, \ldots, x^H) = W(x),$$

where the function W depends on p.

In the *named goods* approach, where «who gets which goods» matters, the domain of the social evaluation problem becomes the space of allocations $q = (q^1, \ldots, q^H)$ in $\mathbb{R}^{L\cdot H}_+$. Under general conditions, Sen (1976) establishes the existence of personalised prices $\pi = (\pi^1, \ldots, \pi^H)$ in $\mathbb{R}^{L\cdot H}_+$, which serve to precipitate a partial social ordering $\mathfrak{R}$ on that space: given two allocations q and r in $\mathbb{R}^{L\cdot H}_+$,

$$\pi q > \pi r \Rightarrow q \,\mathfrak{R}\, r.$$

Introducing a monotonicity condition, Herrero and Villar (1989) extend $\mathfrak{R}$ into a complete ordering. In a competitive context, if the social marginal rate of substitution between any pair of goods for an individual coincides with her private rate of substitution between these goods at market prices p, the ordering $\mathfrak{R}$ can be represented by and additive SEF defined in income space. For each h, let x^h and y^h be the incomes associated, respectively, to the allocations q and r, that is, let

[2] For most purposes in this section we could work with a social ordering. However, since for practical applications it is convenient to use representable orderings, we assume the existence of a SEF from the beginning.

$$x^h = p\, q^h, y^h = p\, r^h,\ h = 1,\dots, H.$$

Then

$$q\, \mathfrak{R}\, r \Leftrightarrow \Sigma_h\, w^h\, x^h \geq \Sigma_h\, w^h\, y^h,$$

where the weights w^h may depend on q, p and H. Therefore, using the direct utility function, we can establish a relation between the SEF S defined in the space of household utilities and that class of additive SEFs defined in income space:

$$S(u) = S[U(q^1),\dots, U(q^H)] = G(q^1,\dots, q^H) = \Sigma_h\, w^h\, x^h.$$

In both approaches, a natural question to ask is: under what conditions it does not matter whether we carry the analysis in income space or, directly, in utility space? In other words, under what conditions on U can we take the functions S and W to be the same or, at least, under what conditions can we be sure that the recommendations by W are congruent with the recommendations by S?

In most welfare economics, social or aggregate welfare is supposed to be related to efficiency and distributional aspects of the problem at hand. In the area of income inequality, distributional considerations are summarized by an inequality index I, which is a real valued function defined on the space of income distributions satisfying a set of minimal properties which will be introduced shortly. In the ethical approach to inequality measurement, one tries to derive inequality indices from specific social evaluation functions in a consistent way. Formally, we say that an inequality measure I is *normatively significant* for –or *consistent* with– a SEF, say W, if for any two distributions x and y with the same mean,

$$I(x) \geq I(y) \Leftrightarrow W(x) \leq W(y).$$

Of course, this is an ordinal property, since the above expression remains satisfied for any monotonic transformation of the I index.

Unfortunately, as Zubiri (1985) demonstrates, due to the nonlinearities of the indirect utility function, income inequality will be a good predictor of utility inequality only under very stringent conditions on both fundamental preferences and admissible inequality measures. In general, we can have an inequality index I_1 normatively significant for S and another I_2 normatively significant for W –not necessarily distinct– and two pairs of distributions (u, x) and (v, y), with $u^h = \varphi\,(x^h, p)$ and $v^h = \varphi\,(y^h, p)$ for each h, such that

$$I_2\,(x) < I_2\,(y) \text{ while } I_1(u) > I_1(v).$$

Therefore, the social evaluation of income distributions at common prices need not lead to the same ranking as the social evaluation of the corresponding utility vectors. This means, in turn, that welfare analysis in income space has to be justified by its own merits.

2.2. *The standard model*

In the ethical approach to inequality measurement we hope that the conditions imposed on the SEF will serve two purposes: that attractive features of the SEF are inherited by the inequality measure, and that these conditions permit to single out a unique inequality index. In the present paradigm, a SEF is always assumed to satisfy symmetry or anonimity and Dalton's principle of progressive transfers, which are best captured by the property of *S-concavity* (assumption A.1 for later reference). This principle states that transfering some income from one agent to another so as to reduce the difference in their welfare should not reduce social welfare[3]. Also, we will assume throughout that the SEF is *continous* (A.2).

In this context, what we call the standard model is characterised by the convenient simplification of making social welfare a function of only two statistics: the mean of the distribution, and an inequality index. Thus, if we denote by μ the function giving the mean, it is worth while to know under what conditions on a SEF there exists an inequality index I and a unique real valued function V defined on $\mathbb{R}^2$ such that

$$W(x) = V[\mu(x), I(x)],$$

with V increasing in its first argument and decreasing in its second argument[4].

There are many ways to derive consistent inequality measures from a given SEF[5]. For later reference, consider the following two well known procedures. Given an income distribution x, define the equally-distributed-equivalent-income (EDEI) ξ as the minimum income that, when assigned to every household, leads to the same social welfare as the original distribution; that is, define $\xi(x)$ implicitly by

$$W(\xi(x),\ldots, \xi(x)) = W(x).$$

If we let $T(x)$ be the total income associated with a distribution x, under assumption A.1. the expression

$$T(x) - H\xi(x)$$

[3] Technically, a SEF W is said to be *S-concave* if for all income vectors x and all doubly stochastic $H x H$ matrix Q we have $W(Qx) \geq W(x)$. The $H x H$ matrix $Q = (q_{ij})$ is said to be doubly stochastic if $q_{ij} \geq 0$, $\Sigma_j q_{ij} = 1$, for all i, j. Since a permutation matrix is a doubly stochastic matrix such that $q_{ij} = 0$ or 1 for all i, j, *S*-concavity implies anonimity.

[4] This problem was originally posed by Kondor (1975).

[5] For the converse question of whether a particular inequality index implies a unique SEF, see Blackorby and Donaldson (1978, 1980) and Ebert (1987).

will be positive except when x is an egalitarian distribution giving the mean to all individuals. Since it provides the amount of total income wasted due to inequality in the distribution x, dividing by $T(x)$ we obtain the proportion of total income wasted due to inequality:

$$I^{AKS}(x) = [T(x) - H\xi(x)] / T(x) = 1 - \xi(x)/\mu(x).$$

Similarly, dividing by H we obtain the *per capita* income wasted due to inequality:

$$I^{KBD}(x) = [T(x) - H\xi(x)] / H = \mu(x) - \xi(x).$$ [6]

These two measures of inequality are normatively significant for W, continous, and S-convex[7].

The problem, as Dutta and Esteban (1992) have shown, is that it is possible to find at least two ordinally different inequality measures which are both normatively significant for a given SEF. This might not be surprising in view of the plethora of indices which could be consistently derived from a given SEF. The practical lesson is that we must go beyond consistency, and be more specific about how to separate changes in total income from distributional changes.

Consider, for instance, the two classes of inequality indices that have received the most attention in the literature. Relative indices are homogeneous of degree zero in incomes, so that an equal proportional change in incomes leaves the level of inequality unchanged. Absolute inequality indices, on the other hand, are invariant to equal absolute changes in individual incomes. Dutta and Esteban (1992) define two inequality indices to be *mean invariant equivalent* if they require additional incomes to be distributed in the same way to maintain the same level of inequality. Thus, for example, all indices of relative inequality are mean invariant equivalent, and so are all indices of absolute inequality. Dutta and Esteban go on to establish that any SEF implies a unique normatively significant inequality measure within any prespecified mean invariance equivalence class[8].

The selection of a mean invariance class of inequality measures imposes a corresponding restriction on the form of the SEF which generates it. Dutta and Esteban (1992) find the nature of this restriction in the general case. As a

[6] The AKS procedure is named after Atkinson (1970), Kolm (1976a) and Sen (1973). The KBD procedure is named after Kolm (1976b) and Blackorby and Donaldson (1980). In addition, Blackorby and Donaldson (1978, 1980, 1984) discuss other procedures to derive consistent inequality indexes from a SEF which depend on a reference welfare level. In this sense, the classical ones presented in the text are reference-free

[7] Bossert and Pifgsten (1990) have proposed a procedure to obtain normatively significant inequality measures that can be conceived as convex combinations of these two polar cases.

[8] This result generalizes those of Blackorby and Donaldson (1978, 1984) and Ebert (1987) on relative inequality indices, and Blackorby and Donaldson (1980) on absolute inequality indices.

corollary, they obtain Ebert (1987) characterisation of the class of *weakly homothetic* SEFs generating relative inequality indices, as well as a characterisation of the class of *weakly translatable* SEFs generating absolute inequality indices[9]. These classes are considerably wider than, respectively, the class of homothetic SEFs and the class of translatable SEFs.

The search for the conditions under which the standard model works is now complete. Let a SEF W satisfy A.1 and A.2, and select a class of mean invariant inequality indices, say, the class of relative measures. Then W can be expressed as a function of the mean and an inequality index of the given class consistent with W if, and only if, W is weakly homothetic (A.3R). Similarly, this is possible for the class of absolute indices if, and only if, W is weakly translatable (A.3A). To ensure that social welfare will be increasing in the mean, an appropriate monotonicity assumption is needed in each case: W must be either *increasing along rays from the origin* in the relative case (A.4R), or *increasing along rays parallel to the line of equality* in the absolute case (A.4A).

To permit comparisons of populations of different size, we will make an assumption, usually refered to as Dalton's *Population Principle* or *replication invariance* (A.5), which says that social welfare is invariant under replications of the population. Since most inequality measures used in practice satisfy this condition, and in the standard model aggregate welfare is a function of only the mean and the inequality of each distribution, it is reasonable to accept A.5.

2.3. *The trade off between efficiency and equity*

Empirical methods have been developed by Shorrocks (1983), Kakwani (1984), Moyes (1987), and Chakravarty (1988) for the unanimous ranking of distributions by all members in wide classes of SEF satisfying the conditions just discussed[10]. A feature of this approach, is that none of these procedures can rank all conceivable distributions. Naturally, whether this lack of completeness poses or not a serious problem has to be judged in each empirical application. On the other hand, the limited interhousehold welfare comparability that these procedures assume, only allows us to conclude that, when the dominance conditions are satisfied, one distribution is prefered by another, but not by how much.

In this paper, we pursue the selection of complete indicators which allow the quantitative decomposition of welfare changes into changes in the mean, and

[9] A SEF W is said to be

I) weakly homothetic if, and only if, for all distributions x and x' such that $\mu(x) = \mu(x')$, $W(x) \geq W(x') \Leftrightarrow W(\alpha x) \geq W(\alpha x')$ for all $\alpha > 0$;

II) weakly translatable if, and only, if, for all distributions x and x' such that $\mu(x) = \mu(x')$, $W(x) \geq W(x') \Leftrightarrow W(x+\lambda 1) \geq W(x'+\lambda 1)$ for all λ such that $(x+\lambda 1), (x'+\lambda 1) \in \mathbb{R}^H_+$.

[10] This literature has been recently enriched in Bishop, Formby and Thistle (1989) by the application of non-parametric statistical procedures for the estimation of the Lorenz curves and related statistics used in this approach.

changes in either relative or absolute inequality. For that purpose, we have to be more specific about the trade off between efficiency and equality. One way to proceed, is simply to select a particular SEF satisfying A.1 to A.5, as in Ebert (1987), which presents an example of a weakly homothetic SEF involving the Gini index of inequality, or as in Shorrocks (1988) who indicates that any index I of relative inequality with the usual properties is normatively significant fot the SEF

$$W(x) = \mu(x) \exp[-I(x)].$$

We may consider also the parametrization suggested by Graaf (1977) and discussed by Atkinson (1989):

$$W(x) = \mu(x)[I(x)]^{\sigma}, \sigma \in [0, 1].$$

However, there are more satisfactory ways of justifying a completeness approach. The following two well known procedures [11] lead to easily interpretable trade offs by imposing assumptions on SEF's which, in turn, allow us to reveal the type of interhousehold welfare comparisons involved. *Homotheticity* (A.6R) of the SEF is necessary and sufficient for the scale-invariance of the inequality index derived according to the AKS procedure which make use of the equally-distributed-equivalent-income concept. Similarly, *translability* (A.6A) of the SEF is necessary and sufficient for the translation-invariance of the inequality index derived according to the KBD procedure. Therefore, any homothetic or translatable SEF can be expressed as a function of the mean and a relative or an absolute inequality index by means of

$$W(x) = \phi(\mu(x)[1 - I^{AKS}(x)])$$

or

$$W(x) = \phi(\mu(x) - I^{KBD}(x)),$$

respectively, where ϕ is an increasing monotonic transformation.

These assumptions imply strong welfare comparability conditions. If the underlying social evaluation ordering is continous, is restricted to the non negative orthant of $\mathbb{R}^H$, and satisfies ratio-scale comparability, then it can be represented by, and only by, a homothetic social evaluation function W. Also, if the social evaluation ordering is continous, and satisfies difference comparability, then it can be represented by, and only by, a translatable social evaluation function W[12].

It should be noticed that the cardinalisation issue, that is, the selection of a function ϕ, is a non trivial matter that must be confronted in practice. The

[11] See, for instance, Blackorby and Donaldson (1978, 1980)

[12] See, for instance, Blackorby and Donaldson (1982) and d'Aspremont (1985, 1994).

connection between the cardinalisation of a SEF and its descomposition by population subgroups, will be treated in the sequel.

2.4. *The distributional role of price changes*

In practice, we are bound to face two or more populations facing different prices. Let x^h_τ be the income of household h in situation τ, with $h = 1_\tau, ..., H_\tau$, and $\tau = 1, 2$. Now we have

$$S(u_1) = S[\varphi(x^1_1, p_1), ..., \varphi(x^H_1, p_1)] = F(x_1, p_1) = W(x_1),$$

where W depends on p_1, and

$$S(u_2) = S[\varphi(x^1_2, p_2), ..., \varphi(x^H_2, p_2)] = F(x_2, p_2) = W(x_2)$$

where W depends on p_2. In general [13], neither of these SEFs can be used to compare the original money distributions x_1 and x_2. However, we can express them at common prices using a true cost-of-living index, say of the Paasche type, defined as follows:

$$P(p_\tau, p_o; u) = c(u, p_\tau) \,/\, c(u, p_o).$$

The function P compares the price vector in situation τ, p_τ, with the vector of base prices p_o at the household utility level u. Individual income in real terms is simply:

$$x^h_{\tau 0} = x^h_\tau \,/\, P(p_\tau, p_o; u^h_\tau) = c(u^h_\tau, p_o).$$

Then, for $\tau = 1,2$,

$$S(u_\tau) = F(x_\tau, p_\tau) = F(x_{\tau 0}, p_o) = W(x_{\tau 0}),$$

so that x_{20} and x_{10} are comparable in terms of the function W which now depends on p_o in both situations.

As we have seen, the income adjustment from x^h_τ to $x^h_{\tau 0}$ causes no welfare change at the individual level. However, at the aggregate level the change from p_τ to p_o produces two effects. To see this, suppose that all households experience as inflationary the change from p_1 to p_o –so that $P(p_1, p_o; u^h_1) > 1$ for every h– but assume that relative prices have evolved less unfavorably for the poor. Then, to maintain their utility levels, the rich must get an income compensation relatively larger than the poor. Thus, on the one hand, we will observe that inequality is larger at x_{10} than at x_1. This is a distributional

[13] See, for instance, Muellbauer (1974a), Roberts (1980), and Blackorby, Laisney and Schmachtenberg (1992).

change worth studying from an ethical point of view. On the other hand, since x^h_{10} is greater than x^h_1 for all h, we will observe that $\mu(x_{10})$ will be greater than $\mu(x_\tau)$, a purely monetary phenomenon without normative significance, which suggests that we should avoid comparisons of income means evaluated at different price vectors. As far as the change in means is concerned, what matters for social evaluations is the comparison between $\mu(x_{20})$ and $\mu(x_{10})$.

Given a SEF W satisfying A.1, A.2, A.4R, A.5 and A.6R, if we adopt the convenient cardinalisation

$$W(x) = \mu(x)\,[1 - I(x)],$$

where I is the index of relative inequality consistent with W, then the welfare change in real terms between situations 1 and 2 can be expressed as

$$W(x_{20}) \,/\, W(x_{10}) = [\mu(x_{20}) \,/\, \mu(x_{10})]\;[[1 - I(x_{20})] \,/\, [1 - I(x_{10})]].$$

The first term is the change in welfare due to the change in the means in real terms. The second term contributes to a welfare improvement if and only if real inequality decreases in the second situation, that is,

$$[1 - I(x_{20})] \,/\, [1 - I(x_{10})] \geq 1 \Leftrightarrow I(x_{20}) \leq I(x_{10}).$$

The welfare impact of a change in real inequality can be decomposed as follows:

$$[1 - I(x_{20})] \,/\, [1 - I(x_{10})] = [1 - I(x_1)] \,/\, [1 - I(x_{10})]\;[1 - I(x_{20})] \,/\, [1 - I(x_2)]\;[1 - I(x_2)] \,/\, [1 - I(x_1)]$$

Thus, for instance, assume that the change in relative prices from p_1 to p_0 involves positive income compensations to sustain situation 1 utility levels, while the change from p_2 to p_0 requires income sustractions to maintain situation 2 utility levels. Assume also that changes are pro-poor in both cases, that is, $I(x_{10}) > I(x_1)$ and $I(x_{20}) < I(x_2)$. Then, the first two terms in the above expression will be greater than one, reflecting a positive welfare contribution. The lesson is that, whatever the change in money inequality, the distributional role of changes in relative prices has an independent impact on the change in real inequality.

Given a SEF W satisfying A.1, A.2. A4A, A.5 and A.6A, if we adopt the convenient cardinalisation

$$W(x) = \mu(x) - A(x),$$

where A is the index of absolute inequality consistent with W, we have

$$W(x_{20}) - W(x_{10}) = [\mu(x_{20}) - \mu(x_{10})] - [A(x_{20}) - A(x_{10})].$$

In this case the change in real welfare is simply equal to the change in means in real terms minus the change in real absolute inequality. Since absolute inequality is not independent of the unit of measurement, it is not useful to decompose the change in real inequality into terms which will contain the effect of the change in relative prices and the change in the general price level.

Naturally, the welfare change in real terms and its decomposition into a change in the mean and a change in relative or absolute inequality, will vary if we reprice the original money distributions at different base prices. A circumstance seldom taken into account in empirical studies [14].

3. The non-homogeneous case

3.1. The domain of the SEF

Let us admit that households are heterogeneous, and let Ã be the set of household characteristics which give rise to ethically relevant differences in needs. Thus, households may now differ in income x^h and/or a vector of characteristics a^h in Ã. There can be a number M of different household types with $1 < M \leq H$. Within each type m, consisting of H^m households, all households have identical characteristics:

$$a^h = a^m \text{ for all } h = 1^m, \ldots, H^m, \; m = 1, \ldots, M$$

If $M = H$, then all households are different. Otherwise, $\Sigma_m H^m = H$.

Since Pollak and Wales (1979), it is generally agreed that, for welfare purposes, households must be endowed with unconditional preferences defined on commodities *and* characteristics, that is, on pairs (q, a) in $\mathbb{R}^L_+ \times \tilde{A}$. If we are realistic and allow households to have different preferences, we would have to find a theoretical justification –and an operational way– to make, at a minimum, interhousehold ordinal level comparisons of the sort

$$u^h = U^h(q^h, a^h) \geq u^k = U^k(q^k, a^k)$$

with (q^h, a^h) not necessarily distinct from (q^k, a^k). In other words, we would need what Pollak (1991) calls a «welfare correspondence» to determine which indifference curve on one households' map yields the same welfare level as a particular curve on each other's map.

Lacking a theory to do that, one way to proceed is by assuming that there exists a common unconditional utility function for all households:

$$u = U^h(q, a) = U(q, a) \text{ for all } h = 1, \ldots, H$$

In terms of these fundamental preferences we can make what Pollak (1991) calls «situational comparisons» of household utilities. To ensure level

[14] For exceptions, see Muellbauer (1974a, b) and the references quoted in Ruiz-Castillo (1993).

comparability of unconditional utilities, for any utility vectors u and v we must have

$$S(u^1,\ldots, u^H) \geq S(v^1,\ldots, v^H) \Leftrightarrow S[\phi(u^1),\ldots, \phi(u^H)] \geq S[\phi(v^1),\ldots, \phi(v^H)],$$

where ϕ is a monotonic increasing function.

If we denote by φ the indirect utility function and by c the cost function, in a given sample of utility maximising and price taking households, the observable data on prices, incomes, characteristics, and commodity demands for each h are related by

$$u^h = U(q^h, a^h) = \varphi(x^h, p, a^h)$$

and

$$x^h = c(u^h, p, a^h).$$ [15]

The conection between utilities, incomes, prices and characteristics is now given by

$$S(u) = S(u^1,\ldots, u^H) = S[\varphi(x^1, p, a^1),\ldots, \varphi(x^H, p, a^H)]$$
$$= F(x^1,\ldots, x^H, p, a^1,\ldots, a^H).$$

In income distribution theory we cannot treat symmetrically the vector of household incomes $x = (x^1,\ldots, x^H)$, each component of which is supposed to serve different needs. There are two approaches to this question. The first one, extends previous results in income distribution dominance to the non-homogeneous case. Atkinson and Bourguignon (1987) start from a SEF in which aggregate welfare is a weighted sum of welfare within each of the subgroups in the partition by ethically relevant characteristics. Subgroup welfare is the sum of the social valuation of income for each household in the subgroup, which is taken to be a function of income and a continuous, one dimensional index n of household need. In our notation,

$$S(u) = \Sigma_m \theta^m S^m(u^{1^m},\ldots, u^{H^m}) = \Sigma_m \theta^m \Sigma_{h=1^m}^{H^m} \varphi(x^h, n^h).$$

Then, conditions are provided for first –and second– degree dominance, which are obtained by making relatively weak assumptions about the relationship between income and needs, i.e., an assumption on the sign of the cross-derivative of income and the index of needs, which implies an ethical ranking of the m household types; and an assumption on higher derivatives which implies that we become less concerned about differences in needs at

[15] In an alternative interpretation devoid of behavioristic features, the fundamental preferences correspond to an agent in charge of aggregate evaluations as in Kolm (1972) or Arrow (1977).

higher incomes, or that the degree of diminishing marginal valuation of income becomes smaller as we move to less needy subgroups [16].

Like in the homogeneous case, the drawbacks of this approach are that the sequential dominance criteria do not provide a complete ordering, and that they do not allow conclusions about how much welfare has improved. In addition, this framework is not suited for the question posed in the following example.

Suppose that total income is equally distributed within two populations, so that each individual receives the mean income of the group, but that the two means are different. For instance, two households of type A and three of type B receive, respectively, (10, 10) and (6, 6, 6) units of income. Under perfect equality, aggregate welfare in each group can be identified with the mean, 10 and 6, respectively. If, for example, type B has more needs, its mean should be corrected and may be reduced to 4. Suppose now that the two groups decide to live together in a single community. Should aggregate welfare in the new community be simply a weighted average of subgroup welfare? If we are interested in welfare decomposition into a within-group and a between-group term, it can be argued that in the new community there is a new source of inequality which should cause a reduction in aggregate welfare below that average, but the income dominance approach is silent on this feature of the situation[17].

The second approach in the literature seeks complete orderings and strong cardinal assertions on inequality and welfare change. The starting point is the adjustment of household incomes using a set of equivalence scales defined in terms of the cost function as follows:

$$d\,(a^h, a^0; p, u) = c\,(u, p, a^h) \;/\; c\,(u, p, a^0).$$

Since the cost function is homogeneous of degree one in prices, the function d is homogeneous of degree zero in p. If we take the reference household a^0 to consist of a single adult, the function d gives the number of equivalent adults in a household of characteristics a^h who can enjoy the utility level u at prices p. For each h in the sample, define the *adjusted,* or *equivalent,* household income by

$$z^h = x^h \;/\; d\,(a^h, a^0; p, u^h) = c\,(u^h, p, a^0)$$

[16] Bourguinon (1989) extends the approach to households that maximise the sum of their members' individual utility. A dominance criterion is then derived under the assumptions that household indirect utility functions are increasing and concave with income, and the marginal utility of income increases with the index of need. Jenkins and Lambert (1993) extend Atkinson and Bourguignon (1987)'s work to situations where the marginal distribution of needs differ. Then a distinction can be made between a welfare-improving income distribution change and a welfare-improving population composition change.

[17] So is Ebert (1993)'s attempt at characterising measures of relative inequality in the non-homogeneous case.

This is the income necessary for a single adult to enjoy the utility level u^h at prices p. Alternatively, we can define the compensation function

$$d^* (a^h, a^0; p, u) = c(u, p, a^h) - c(u, p, a^0)$$

which is homogeneous of degree one in p. It gives the income we can subtract from a household of characteristics a^h for a single adult to enjoy the same utility level u at prices p with the remaining income. Then

$$z^h = x^h - d^* (a^h, a^0; p, u^h) = c(u^h, p, a^0)$$

In the important case of comparisons between two heterogeneous populations confronting different price vectors in situations $\tau = 1, 2$, equivalent household income would be:

$$z^h_{\tau 0} = x^h_t / [P(p, p_o; u^h_\tau, a^h_\tau)\, d(a^h_\tau, a^0; p_o, u^h_\tau)]$$

$$= [x^h_\tau / P(p, p_o; u^h_\tau, a^h_\tau)] - d^* (a^h_\tau, a^0; p_o, u^h_\tau) = c(u^h_\tau, p_o, a^0)$$

Of course, for each h we have

$$u^h_\tau = \varphi(x^h_\tau, p_\tau, a^h_\tau) = \varphi(z^h_{\tau 0}, p_o, a^0),$$

while for every pair of households h, k, we have

$$z^h_{\tau 0} \geq z^k_{\tau 0} \Leftrightarrow c(u^h_\tau, p_o, a^0) \geq c(u^k_\tau, p_o, a^0) \Leftrightarrow u^h_\tau \geq u^k_\tau;$$

that is, income adjusted for price change and non-income needs provides a comparable indicator of household welfare.

This fact provides good conceptual and political reasons for worrying about aggregation in adjusted income space. The social evaluation problem can be finally stated as the comparison between $W(x_{20})$ and $W(x_{10})$, with

$$S(u_\tau) = F(x_\tau, p_\tau, a_\tau) = F(z_{\tau 0}, p_o, a^0) = W(z_{\tau 0}), \tau = 1, 2,$$

where the function W depends on both p_o and a^0. Since adjusted incomes are now comparable, W can now be taken to be symmetric, as implied in the standard model when we assume W to be S-concave.

Before ending this discussion, it is important to realize how Daltonian transfers should be implemented in this context. We suggest to define progressive (regressive) transfers of equivalent income from a household h to a poorer (richer) household k, so as to leave unchanged both the ranking between h and k, as well as total equivalent income. However, in the absence of restrictions on fundamental preferences, transfers which preserve the mean of equivalent incomes will usually require changes in the mean of unadjusted incomes. Nevertheless, this should cause no particular concern, since we know that the latter is not the distribution ethically relevant in the non-homogeneous case [18].

3.2. *Welfare decomposition by population subgroup*

For any partition of the population, we are interested in welfare measures capable of distinguishing –in a convenient additive way– between two components: welfare within the subgroups, and the loss of welfare due to inequality between the subgroups. To achieve this practical aim, we will have to impose further restrictions on SEFs. As we will see, these restrictions will be final, because we will end up with a single SEF for the relative case, and a family of translatable SEFs for the absolute case.

Without loss of generality, we will consider unweighted distributions of equivalent income. Although the results presented in this section are valid for any partition of the population, for illustrative purposes we will always refer to the partition according to the ethically relevant household characteristics. There are different ways to decompose a summary statistic like an inequality or a welfare index. The following two are the best known.

Method I

Consider the three distributions:

$$z = (z^1, \ldots, z^M) \qquad [1]$$

$$\mu^* = (\mu^1, \ldots, \mu^M) \qquad [2]$$

$$\mu = (\mu(z).1^H), \qquad [3]$$

where for each m,

$$\mu^m = (\mu(z^m).\ 1^{H^m}), \text{ and } 1^{H^m} = (1, \ldots, 1) \in \mathbb{R}^{H^m};$$

[1] is the adjusted income distribution; in [2] there is no within group inequality because each household has been assigned her subgroup mean income $\mu(z^m)$, and in [3] there is no inequality because each household receives the population mean $\mu(z)$. In Method I, between-group inequality is defined as the inequality that would arise in a movement from [3] to [2]. Then, one investigates under what conditions overall inequality can be expressed as

$$I(z) = \Sigma_m \alpha^m I(z^m) + I(\mu^*), \qquad [4]$$

[18] Contrast this position with Glewwe (1991)'s discussion of an example in which a regressive transfer in unadjusted incomes caused an increase in the mean of the adjusted distribution after the transfer, altering the relative share of every one and giving rise to an improvement in the inequality of adjusted incomes. Ebert (1993) attempts to work in both spaces in a consistent way, rather than concentrating, as we suggest, on equivalent incomes space.

where the weights α^m are functions only of the set of subgroup means and sizes. If we now choose a cardinalisation of a SEF W such that

$$W(z) = \mu(z)\,[1 - I(z)]$$

and subgroup weights in equation [4] are subgroup shares in total income, that is,

$$\alpha^m = T(z^m) \,/\, T(z),$$

then we will have

$$W(z) = \Sigma_m\,[H^m / H]\; W(z^m) - \mu(z)\, I(\mu^*).$$

This is a useful expression –to which we will refer as A.7I– that indicates that aggregate welfare can be expressed as a weighted average of the welfare within each subgroup, with weights equal to population shares, minus the between-group inequality according to Method I, weighted by the population mean.

It is well known [19] that an index of relative inequality has continous first order derivatives, satisfies S-convexity and the Population Principle, and admits the decomposition of equation [4] if, and only if, it is an increasing monotonic transformation of an index from the generalised entropy family:

$$I_c(z) = [1 / Hc\,(c-1)]\; \Sigma_h\,[[z^h / \mu(z)]^c - 1],\; c \in \mathbb{R},\; c \neq 0, 1,$$
$$I_0(z) = [1 / H]\; \Sigma_h\,[\ln \mu(z) - \ln z^h], \quad c = 0,$$
$$I_1(z) = [1 / H]\; \Sigma_h\,[z^h / \mu(z)]\; \ln\,[z^h / \mu(z)], \quad c = 1$$

The weights in the decomposition are equal to

$$\alpha^m_c = H^m / H\,[\mu(z^m) / \mu(z)]^c,$$

so that $\Sigma_m\, \alpha^m_c = 1$ only if $c = 0$ or 1. Therefore, for the class of SEF

$$W_c(z) = \mu(z)\,[1 - I_c(z)], \qquad [5]$$

we will have

$$W_c(z) = \Sigma_m\, \beta^m_c\, W_c(z^m) - \mu(z)\, I_c(\mu^*),$$

where

$$\beta^m_c = [H^m / H]\; [\mu(z^m) / \mu(z)]^{c-1}$$

[19] See, for instance, Cowell (1980), Shorrocks (1980), or the illuminating discussion in Shorrocks (1988).

Notice that $\Sigma_m \beta^m_c = 1$ only if $c = 1$ or 2, in which case $\beta^m_1 = H^m / H$, and $\beta^m_2 = T(z^m) / T(z)$, respectively. Clearly, from a normative point of view demographic weights are to be prefered to income shares, so that we should choose the SEF

$$W_1(z) = \mu(z)[1 - I_1(z)] = \Sigma_m [H^m / H] W_1(z^m) - \mu(z) I_1(\mu^*) \qquad [6]$$

which is the only member of the entropy family which satisfies A.7I.

Moreover, following a suggestion in Herrero and Villar (1989), the function W_1 can be rationalised within the *named goods* approach reviewed in the first section. There we saw that

$$W(z) = \Sigma_h w^h z^h,$$

where the weights w^h may depend on the allocation of commodities to households, market prices, and population size. Let us make

$$w^h = [1 - ln(z^h / \mu(z)] / H,$$

so that households whose income equals the population mean receive a weight equal to $1 / H$, and households with income above or below the mean receive weights increasingly smaller or greater, respectively, than $1 / H$. Then it can be shown that the function W becomes W_1.

As pointed out in Tomás and Villar (1993), since the index I_1 can vary in the interval $[0, lnH]$, for inequality values greater than 1 welfare will vary negatively with income. These authors solve such normalization problem in the context of an axiomatic characterization of the weights

$$w^{*h} = [1 - ln(z^h / \mu(z)] / H\, lnH$$

which yield the SEF

$$W^*(z) = \Sigma_h w^{*h} z^h = \mu(z)[1 - [I_1(z) / lnH]]$$
$$= \Sigma_m [H^m / H] \mu(z^m)[1 - [I_1(z^m) / lnH]] - \mu(z) I_1(\mu^*) / lnH,$$

whose decomposition is not as nice as the corresponding to the non-normalized version W_1.

Method II

Blackorby, Donaldson and Auersperg (1981) suggest a different conceptual experiment. They study ethical indices of inequality derived from a SEF by the AKS or the KBD procedure, and consider the two distributions

$$\xi^* = (\xi^1,..., M) \quad [7]$$
$$\xi = (\xi\,(z).\ 1^H), \quad [8]$$

where for each m,

$$\xi^m = (\xi\,(z^m).\ 1^{H^m}).$$

In [7] there is no within group inequality, while in [8] there is no inequality at all. In Method II, the between-group component of inequality is defined as the inequality that would result if each household received her subgroup's EDEI $\xi\ (z^m)$; that is, as the inequality produced in going from [8] to [7], measured as the percentage of total income wasted in the AKS case, or the *per capita* loss in the KBD case. Similarly, the withingroup component will be the percentage of total income wasted, or the per capita loss, in the movement from [7] to the original distribution.

In Method II all distributions have the same level of social welfare, $W(z) = W(\xi^*) = W(\xi)$, while in Method I $W(z)$, $W(\mu^*)$, and $W(\mu)$ are all different. Thus, Blackorby *et al* indicate that their intergroup inequality index measures differences in the *actual* economic positions of the subgroups rather than their potential positions at the mean of the corresponding distributions. The difference between the two approaches can be seen in an example of a population of two males with incomes (10,10), and two females with incomes (18, 2). Since both subgroup means are 10, Method I would measure between-group inequality as zero. Using instead the EDEIs to eliminate the inequality within each subgroup, Method II would generate positive intergroup inequality, reflecting the fact that in this example the sexes do not receive equal treatment.

Naturally, one could construct an example in which $\xi^1 = \xi^2$ but $\mu^1 \neq \mu^2$, with the implications reversed.

To apply Method II, for any partition subgroup inequality must be independent on outside incomes. If W is continous, increasing, and S-concave, Blackorby *et al* established that this minimal condition leads to the additive separability of W, i. e.

$$W(z) = \phi\ [\Sigma_h \Psi(z^h)],$$

where ϕ is increasing in its argument and Ψ is concave. We will refer to this condition as assumption A.7II. If, in addition, W is translatable, then it can only be an increasing transformation of the Kolm-Pollak family:

$$W_\gamma(z) = -[1/\gamma]\ 1n\ [(1/H)\ \Sigma_h e^{-\gamma z^h}],\ \gamma>0,$$

where γ is interpreted as an aversion to inequality parameter: as γ increases, the social indifference curves show increasing curvature until only the income of the poorest person matters. The KBD index of absolute inequality consistent with the simplest possible cardinalization of W_γ, or the Kolm-Pollak index, is

$$A_\gamma(z) = [1/\gamma] \ln [(1/H)\Sigma_h e^{\gamma(\mu(z) - z^h)}], \gamma>0.$$

Since

$$A_\gamma(z) = \Sigma_m [H^m/H] A_\gamma(z^m) + A_\gamma(\xi^*),$$

we have

$$W_\gamma(z) = \mu(z) - A_\gamma(z) = \Sigma_m [H^m/H] W_\gamma(z^m) - A_\gamma(\xi^*). \quad [9]$$

This is an appealing decomposition, in which social welfare is seen to be equal to the weighted average of the aggregate welfare within each of the subgroups, with weights equal to population shares, minus the inequality between the subgroups according to Method II.

When all incomes are restricted to be positive and W is homothetic, continous, increasing, S-concave, and additively separable, then W becomes an increasing transformation of the family of means of order r:

$$W_r(z) = [1/H] \Sigma_h (z^h)^r]^{1/r}, r \neq 0, r \leq 1$$
$$\Pi_h (z^h)^{1/H}, r = 0.$$

The AKS index of relative inequality consistent with the simplest cardinalisation of W_r is the Atkinson family

$$I_r(z) = 1 - [1/H] \Sigma_h (z^h/\mu(z))^r]^{1/r}, r \neq 0, r < 1$$
$$1 - \Pi_h (z^h/\mu(z))^{1/H}, r = 0.$$

The problem is that the index I_r does not admit a convenient decomposition into a within-group term and a between-group term and, therefore, neither does W_r. However, this family is ordinally equivalent to part of the entropy family. As a matter of fact, for the range $c = r < 1$, we have

$$I_r(z) = [[c^2 - c] I_c(z) + 1]^{1/r}$$

and

$$I_c(z) = 1 - [[1 - I_r(z)]^r - 1]/(c^2 - c). \quad [10]$$

As pointed out by Cowell and Jenkins (1994), the question of the between-group inequality definition is distinct from the cardinalisation issue. Therefore, the mean-income decomposition characteristic of the entropy family could be applied to an inequality measure that is in some other cardinalisation. Thus, an Atkinson-type measure could be decomposed according to the mean-income decomposition by applying the transformation [10] to I_c in an expression like [5]. Unfortunately, the case $c = 1$ for which assumption A.7I is satisfied is outside of this suggestion's range of applicability, so that in the relative case we simply recommend using directly the SEF W_1 obtained via Method I.

3.3. A useful restriction on household preferences

As pointed out in Muellbauer (1974a), in the non-homogeneous case we must face a second index number problem. The conversion of the income distributions x_τ into the adjusted or equivalent distributions z_τ for $\tau = 1, 2$, is conditional on a reference vector of characteristics a^0. If such reference vector varies, so will the values of $W(z_\tau)$ and, therefore, the empirical results for welfare change.

A particular aspect of such a problem deserves closer attention. Let us consider a homogeneous subgroup of the population sharing the vector of characteristics a^m. Households in this group have the same needs, but identical characteristics might be enjoyed differently depending on the income level. For instance, identical households might experience different economies of scale in consuption depending on their income level. Ignoring common prices, this phenomenon is captured in the functions $d(a^m, a^0; u^h)$ and $d^*(a^m, a^0; u^h)$, which depend on household welfare.

At the individual level, the switch from a^m to a^0 does not alter households utility levels. However, at the aggregate level we have two effects. Because the rich take advantage of economies of scale at different intensities than the poor, both relative and absolute inequality of the distribution x^m will differ from the inequality of equivalent income z^m. Just as the impact of changing relative prices from a given price vector to base prices varies with the reference price vector, whether we choose a single adult or a couple as reference type will have empirical consequences for the measurement of inequality within homogeneous subgroups.

On the other hand, there is a scale effect which will affect both the mean and the EDEI of the adjusted distribution. For example, if for all households in subgroup m $d(a^m, a^0; u^h) > 1$ –or $d^*(a^m, a^0; u^h) \neq 0$– we will have that $\mu(z^m) < \mu(x^m)$ and whenever we use an homothetic or translatable SEF, $\zeta(z^m) < \zeta(x^m)$. This effect resembles the pure monetary effect induced by the adjustment of money incomes to changes in the general price level and, consequently, it lacks normative significance for each group in isolation. However, since its magnitude will vary across groups, it will have an impact on the measurement of overall inequality and welfare. Such an impact will depend on the value judgement implicit in the choice of reference type.

By the combined influence of these two forces, every homogeneous subgroup's welfare depends on that choice. Thus, for example, if the adjustment process is pro-rich, i.e. if $d(a^m, a^0; u^h)$ or $d^*(a^m, a^0; u^h)$ are smaller for the rich, then $I(z^m)>I(x^m)$ or $A(z^m)>A(x^m)$ in the relative or the absolute case. Hence, $W(z^m) < W(x^m)$. Otherwise, if the adjustment process is pro-poor, then the improvement in inequality of equivalent income may compensate the reduction of the mean. In either case, the within-group welfare term in adjusted income space will bear a complex relationship with the same term for unadjusted incomes, and the difference between these two concepts will vary depending on the reference type.

On the other hand, because of the scale effect, in both the relative and the absolute case the between-group component of inequality will change with the reference type and will be different from that term for the unadjusted distribution. Thus, both terms in the decomposition of overall welfare given in equations [6] and [9], will vary with the reference type and the form of the fundamental cost function which determines the way in which the functions d and d^* vary with utility levels.

It seems worthwhile to consider the special case in which the extent of the economies of scale in consumption are the same independently of the level of unadjusted income or, more generally, the case in which the functions d and d^* are independent of utility levels. Of course, this will restrict household preferences. The function d is independent of utility levels –condition IB (Independent of Base utility) in Lewbel (1989), or EES (Equivalence-Scale Exactness) in Blackorby and Donaldson (1989)– if, and only if, the cost function adopts the following form, which we will refer to as assumption A.8R:

$$c(u,p,a) = f(u,p)g(p,a).$$

As Blackorby and Donaldson (1989) show, this restriction on preferences is equivalent to a condition on interhousehold comparisons which requires that, if there exist incomes such that the members of two households facing the same prices are equally well-off, then any scaling of household incomes preserves equality of well-being. It is a normalization on income consumption curves which, however, does not require them to be straight lines.

Similarly, in the absolute case Blackorby and Donaldson (1994) show that the function d^* is independent of the utility level if, and only if, the cost function becomes

$$c(u,p,a) = f(u,p) + g(p,a).$$

This assumption A.8A implies also a corresponding restriction on interhousehold comparisons or a normalization on income-consumption curves.

Under A.8R and A.8A, equivalent incomes in a given subgroup m will differ from unadjusted incomes in a constant –either $D(a^m, a^0)$ or $D^*(a^m, a^0)$– depending only on the reference type. Therefore, at constant prices, relative and absolute inequality before and after the adjustment will be the same independently of the reference type a^0:

$$I(z^m) = I(x^{1^m}/D(a^m, a^0),\ldots, x^{H^m}/D(a^m, a^0)) = I(x^m)$$

if I is an index of relative inequality, and

$$A(z^m) = A(x^{1^m} - D^*(a^m, a^0),\ldots, x^{H^m} - D^*(a^m, a^0)) = A(z^m)$$

if A is an index of absolute inequality.

As far as the effect on the subgroup means, in the relative case we have

$$\mu(z^m) = \mu(x^m)/D(a^m, a^0).$$

Therefore, if W is homothetic, then for each m

$$W(z^m) = \mu(z^m)[1-I(z^m)] = \mu(x^m)[1-I(x^m)]/D(a^m, a^0) = W(x^m)/D(a^m, a^0).$$

Since, in addition,

$$\mu(z) = \Sigma_m [H^m/H]\mu(z^m),$$

it can be shown that equation [6] for the Theil welfare measure becomes

$$W_1(z) = \Sigma_m [H^m/H][W_1(x^m)/D(a^m, a^0)] - \mu(z)I_1(\mu^*).$$

The within-group term is the weighted average of subgroup welfare of unadjusted incomes, scaled by the factor $D(a^m, a^0)$. Of course, the impact of the choice of a^0 will be reflected also in the second term of the decomposition.

In the absolute case,

$$\zeta(z^m) = \zeta(x^m) - D^*(a^m, a^0) \text{ and } \mu(z^m) = \mu(x^m) - D^*(a^m, a^0).$$

If W is translatable, then for each m

$$W(z^m) = \mu(z^m) - A(z^m) = \mu(x^m) - D^*(a^m, a^0) - A(x^m) = W(x^m) - D^*(a^m, a^0).$$

Thus, equation [9] for the Kolm-Pollak family becomes

$$W_\gamma(z) = \Sigma_m [H^m/H]\, W_\gamma(x^m) - \Sigma_m [H^m/H]\, D^*(a^m, a^0) - A_\gamma(\zeta^*).$$

Within-group welfare is independent of the reference type, whose choice only affects overall welfare through a demographic term and between-group inequality.

3.4. *The weighting of households for social evaluation*

It should be clear that the assumption of a common utility function, defined on household consumption and characteristics, only allows us to compare the *household* welfare of households of different characteristics. That is, it only allows us to perform inter-household, not inter-personal, welfare comparisons.

This presents a problem, since in welfare economics we are mostly interested in personal welfare. Budget surveys contain little information on how household income or expenditure is shared by persons inside each household. Also, recent developments of testable theories on this allocation problem, although very promising, have not yet reached suitable results for regular use in income distribution analysis [20]. Surely, without better data it is hard to see how to make in practice inter-personal comparisons of utility both within and across households.

However, without abandoning the present framework, we can still ask whether we want to count all households equally. There is no difficulty in extending the domain of the SEF, weighting each household h by an scalar β^h, provided the procedure receives an adequate justification. To begin with, lacking ethical reasons to discriminate within types, we will restrict ourselves to the case in which all households of the same type m, receive the same weight β^m.

Next, recall that, in the abscence of restrictions on fundamental preferences, we saw that transfers which preserve the mean of equivalent incomes will usually require changes in the mean of unadjusted incomes. Under A.8R we have that this will not be the case if, and only if,

$$\beta^h / D(a^h, a^0) = \beta^k / D(a^k, a^0).$$

Similarly, under A.8A, transfers which preserve the mean of equivalent incomes will not require changes in the mean of unadjusted incomes if, and only if, $\beta^h = \beta^k$. Hence, both in the relative and the absolute case, transfers of equivalent income between households of the same subgroup preserve the mean of the unadjusted incomes.

However, when the transfer involves households from different subgroups the situation is as follows. In the relative case, if the weights are the number of equivalent adults, i.e., $\beta^h = D(a^h, a^0)$ for all h, then the mean of the unadjusted incomes is preserved. This is the main reason why Ebert (1992) recommends this weighting scheme. In the absolute case, total unadjusted

[20] See, for instance, the excellent survey by Bergstrom (1993).

income will be constant after the transfer only in the unweighted case. But this does not seem to be a sufficient reason to single out these two particular procedures. As a matter of fact, in empirical applications in both the relative and the absolute case, we recommend experimenting with different weighting schemes, regardless of the impact of equivalent income transfers on the unadjusted distribution.

For that purpose, we give here the expression for welfare measurement under A.8R and A.8A for the weight choices most often used in practice. Let $y = (y^1,..., y^M)$ be the distribution where adjusted incomes are weighted by the number of household members s^h[21], and assume that all households of type m have the same number s^m. Then, given A.5, in the relative case we have

$$I(y^m) = I[s^m(x^{1m} / D(a^m, a^0)),..., s^m(x^{Hm} / D(a^m, a^0))] = I(x^m),$$

and

$$\mu(y) = \Sigma_h [S^m / S] \mu(y^m) = \Sigma_h [S^m / S] \mu(x^m) / D(a^m, a^0),$$

where $S^m = mH^m$ is the number of persons of type m, and $S = \Sigma_m S^m$. Hence,

$$W_1(z) = \Sigma_m [S^m / S] [W_1(x^m) / D(a^m, a^0)] - \mu(y) I_1(\mu_S^*),$$

where

$$\mu_S^* = (\mu_S^1,..., \mu_S^M), \mu_S^m = (\mu(z^m). 1^{Sm}), \text{ and } 1^{Sm} = (1,..., 1) \in R^{Sm}$$

Similarly, in the absolute case we have

$$A_\gamma(y^m) = A_\gamma[s^m(x^{1m} - D^*(a^m, a^0)),..., s^m(x^{Hm} - D^*(a^m, a^0))] = A_\gamma(x^m),$$

and

$$\mu(y^m) = \mu(x^m) - D^*(a^m, a^0).$$

Hence,

$$W_\gamma(z) = \Sigma_m [S^m / S] W_\gamma(x^m) - \Sigma_m [S^m / S] D^*(a^m, a^0) - A_\gamma(\zeta_S^*),$$

where

$$\zeta_S^* = (\zeta_S^1,..., \zeta_S^M), \zeta_S^m = (\zeta(z^m). 1^{Sm})$$

Finally, for the distribution $e = (e^1,..., e^M)$ where adjusted incomes are weighted by the number of equivalent adults, in the relative case we have

[21] This is the weighting scheme recommended by Dazinger and Taussig (1979) and Cowell (1984), and used by the present author in Ruiz-Castillo (1993, 1994).

$$I(e^m) = I(x^m), \text{ and } \mu(e) = \mu(x^m)$$

Let $E^m = D(a^m, a^0) H^m$ be the number of equivalent adults of type m, and $E = \Sigma_m E^m$. Then,

$$W_1(e) = \Sigma_m [E^m / E] [W_1(x^m) / D(a^m, a^0)] - \mu(e) I_1(\mu_e^*),$$

where

$$\mu_e^* = (\mu_e^1, ..., \mu_e^M), \mu_e^m = (\mu(z^m). 1^{E^m}), \text{ and } 1^{E^m} = (1, ..., 1) \in \mathbb{R}^{E^m}.$$

4. Conclusions

We have presented a complete framework of analysis to make «situational comparisons» in the sense of Pollak (1991) of independent cross-sections of household income and non-income household characteristics. Making use of the indirect utility function, we have been able to examine the relationship between utility levels and observable variables like income, prices and characteristics. Under general conditions, due to the non-linearities of the indirect utility function, we cannot guarantee that welfare recommendations in income space will lead to normatively consistent recommendations in utility space, even in the simplest case of a homogeneous population. Hence, income distribution theory needs a different justification.

To make any progress at all in the non-homogeneous case, we can follow two routes. Firstly, we can assume an additive structure where social welfare is a weighted average of subgroup welfare, which, in turn, is the sum of the social valuation of income and an index of needs for each household in the subgroup. Then, stochastic dominance criteria, coupled with relatively weak assumptions on the relationship between income and needs, permit an incomplete ordering of income distributions for a heterogeneous population.

Secondly, based on the existence of a fundamental utility function, adjusted income for price change and ethically relevant non-income needs, provides a comparable household welfare indicator. With an operational aim in mind, we proceed to ask which properties should be required from a SEF defined on adjusted or equivalent income space beyond ordinal level comparability.

Consider the standard model where, given a SEF W, there exists a unique function V such that

$$W(z) = V(\mu(z), I(z)),$$

where μ is the mean, I is an inequality measure normatively significant for W, and V is increasing in its first argument and decreasing in its second argument. The following is a minimal set of axioms for W characterising this model in the two polar cases for relative and absolute inequality measures:

A.1. S-concavity

A.2. Continuity

A.3R. Weak-homotheticity and A.4R. Monotonicity along rays from the origin, if I is an index of relative inequality.

A.3A. Weak-translability and A.4A. Monotonicity along rays paralell to the line of equality, if I is an index of absolute inequality.

A.5. Invariance to population replication.

Suppose we are interested in a complete ordering and a cardinal interpretation of changes in aggregate welfare, which permits a quantitative decomposition real welfare changes into a change in means at constant prices and a change in real inequality. Then, in the two polar cases, we may impose on W:

A.6R. Homotheticity

or

A.6A. Translability

Suppose that, in addition, we are interested in welfare decomposition by population subgroups in terms of a within-group and a between-group component in the partition according to ethically relevant (and other) characteristics. In the relative case, consider

$$\text{A.7I. } W(z) = \Sigma_m \, [H_m \, / \, H] \; W(z^m) - \mu \, (z) \, I \, (\mu^*),$$

where μ^* is the distribution in which each household receives her subgroup mean. Under an appropiate cardinalisation, assumptions A.1, A.2, A.4R, A.5, A.6R and A.7I are satisfied in the differentiable case if, and only if, W is

$$W_1 \, (z) = \Sigma_m \, [H_m \, / \, H] \; W_1 \, (z^m) - \mu \, (z) \, I_1 \, (\mu^*),$$

where I_1 is Theil's first inequality index. Consider a separability condition for computing any subgroup EDEI independently of the rest of the population

$$\text{A.7II. } W(z) = \phi \, [\Sigma_h \, \psi \, (z^h)],$$

where ϕ is increasing and ψ concave. In the absolute case, A.1, A.2, A.4A, A.5, A.6A and A.7II are satisfied, if, and only if, W is the Kolm-Pollak family $W_\gamma \, (z)$ with $\gamma > 0$ and, under an appropiate cardinalisation,

$$W_\gamma \, (z) = \Sigma_m \, [H^m \, / \, H] \; W_\gamma \, (z^m) - A_\gamma \, (\zeta^*),$$

where ξ^* is the distribution in which each household receives her subgroup EDEI.

In the abscence of further restrictions on unconditional preferences, inequality within each homogeneous subgroup depends on the value judgement implied in the choice of a reference type. To avoid this, and to expose the impact of such choice on the within-group and between-group terms of the ethically relevant partition, we may assume that the adjustment procedure for taking into account non-income needs is independent of the utility level. Then, in the relative case we must assume

$$\text{A.8R. } c(u, p, a) = f(u, p)\, g(p, a),$$

while in the absolute case

$$\text{A.8A. } c(u, p, a) = f(u, p) + g(p, a).$$

Under A.8R and A.8A,

$$W_1(z) = \Sigma_m [H^m / H]\, [W_1(x^m) / D(a^m, a^0)] - \mu(z)\, I_1(\mu^*),$$

where

$$\mu(z^m) = \mu(x^m) / D(a^m, a^0),$$

and

$$W_\gamma(z) = \Sigma_m [H^m / H]\, W_\gamma(x^m) - \Sigma_m [H^m / H]\, D^*(a^m, a^0) - A_\gamma(\zeta^*),$$

where

$$\zeta(z^m) = \zeta(x^m) - D^*(a^m, a^0).$$

Extensions of the SEF's domain by means of weighting schemes which allow households to count in proportion to their size, can be easily handled in this framework.

References

Amiel, Y. and Cowell, F. A. (1992): «Measurement of Income Inequality. Experimental Test by Questionnaire», *Journal of Public Economics* 47, pp. 3-26.

Arrow, K. (1977): «Extended Sympathy and the Possibility of Social Choice», *American Economic Review, Papers and Proceedings,* (February): pp. 219-225

Atkinson, A. (1970): «On the Measurement of Inequality» *Journal of Economic Theory* 2: pp. 244-263.

Atkinson, A. B. (1989): «Measuring Inequality and Differing Value Judgements», ESRC Programme on Taxation, Incentives, and the Distribution of Income, Discussion Paper, 129.

Atkinson, A. B. and Bourguignon F. (1987): «Income Distribution and Differences in Need», in G. R. Feiwell (ed), *Arrow and the Foundations of the Theory of Economic Policy*, London: Macmillan.

Ballano, C. and Ruiz-Castillo, J. (1993): «Searching by Questionnaire for the Meaning of Income Inequality», *Revista Española de Economía* 10, pp. 233-259.

Bergstrom, T. (1993): «A survey of Theories of the Family», University of Michigan, mimeo.

Bishop, J.; Formby, J. and Thistle, P. (1989): «Statistical Inference, Income Distributions, and Social Welfare», in D. J. Slotje (ed), *Research on Economic Inequality*, Vol. I, Greenwich, CT: Jay Press.

Blackorby, C. and Donaldson, D. (1978): «Measures of Relative Inequality and their Meaning in Terms of Social Welfare», *Journal of Economic Theory* 18, pp. 59-80.

Blackorby, C. and Donaldson, D. (1980): «A Theoretical Treatment of Indices of Absolute Inequality», *International Economic Review* 21, pp. 107-136.

Blackorby, C. and Donaldson, D. (1982): «Ratio-scale and Translation-scale Full Interpersonal Comparability Without Domain Restrictions: Admissible Social Evaluation Functions», *International Economic Review* 23, pp. 249-268.

Blackorby, C. and Donaldson, D. (1984): «Ethically Significant Ordinal Indices of Relative Inequality», in G. Rhodes and R. Basmann (ed), *Advances in Econometrics*, Vol. III, Greenwich: JAI Press.

Blackorby, C. and Donaldson, D. (1993): «Adult Equivalence Scales and the Economic Implementation of Interpersonal Comparisons of Well-Being», *Social Choice and Welfare* 10, pp. 335-361.

Blackorby, C. and Donaldson, D. (1994): «Measuring the Cost of Children», in I. P. R. Blundell I. Walker (ed), *The Measurement of Household Welfare*, Cambridge: Cambrige University Press.

Blackorby, C.; Donaldson, D. and Auersperg, M. (1981): «A New Procedure for the Measurement of Inequality Within and Among Population Subgroups», *Canadian Journal of Economics* 14, pp. 665-685.

Blackorby, C.; Laisney, F. and Schmachtenberg, R. (1992): «Reference-Price-Independent Welfare Prescriptions», *Journal of Public Economics* 50, pp. 63-76.

Bossert, W. and Pfingsten, A. (1989): «Intermediate Inequality: Concepts, Indices, and Welfare Implications», *Mathematical Social Sciences* 19, pp. 117-134.

Bourguignon, F. (1989): «Family Size and Social Utility», *Journal of Econometrics* 42, pp. 67-80.

Chakravarty, S. (1988): *On Quasi-Orderings of Income Profiles*, University of Paderborn, Methods of Operations Research 60, XIII.

Cowell, F. A. (1980): «On the Structure of Additive Inequality Measures», *Review of Economic Studies* 47, pp. 521-531.

Cowell, F. (1984): «The Structure of American Income Inequality», *Review of Income and Wealth* 30, pp. 351-375.

Cowell, F. and S. Jenkins (1994): *How Much Inequality Can We Explain? A Methodology and an Application to the USA,* Distributional Analysis Research Programme, London School of Economics, Discussion Paper, No. DARP-7

Dalton, H. (1920): «The Measurement of Inequality of Income», *Economic Journal* 30. pp. 348-361.

Deaton, A. and Muellbauer, J. (1980): *Economics and Consumer Behavior.* Cambridge: Cambridge University Press.

Danziger, S. and Taussing, M. (1979): «The Income Unit and the Anatomy of the Income Distribution», *Review of Income And Wealth* 79, pp. 365-375.

D'Aspremont, C. (1985): «Axioms for Social Welfare Choice», in L. Hurwicz, D. Smeidler and H. Sonnenshein (ed), *Social Goals and Social Organization. Essays in Honor of Elisha Pazner,* Cambridge: Cambridge University Press.

D'Aspremont (1994): «Welfarism and Interpersonal Comparisons», *Investigaciones Económicas* 18, pp. 3-17.

Deaton, A. and Muellbauer, J. (1980): *Economics and Consumer Behavior.* Cambrigde: Cambridge University Press.

Dutta, B. and Esteban, J. M. (1992): «Social Welfare and Equality», *Social Choice and Welfare* 50, pp. 49-68.

Ebert, U. (1987): «Size and Distribution of Incomes as Determinants of Social Welfare», *Journal of Economic Theory* 41, pp. 23-33.

Ebert, U. (1992): *On Comparisons of Income Distributions When Household Types Are Different,* Department of Economics, University of Oldenburg, V–86-92.

Ebert, U. (1993): *Income Inequality and Differences in Household Size,* Department of Economics, University of Oldenburg, V-83-92.

Glewwe, P. (1991): «Household Equivalence Scales and the Measurement of Inequality: Transfers from the Poor to the Rich Can Decrease Inequality», *Journal of Public Economics* 44, pp. 211-216.

Graaff, J. d. V. (1977): «Equity and Efficiency as Components of the General Welfare» *South African Journal of Economics* 45. pp. 362-375.

Harrison, E. and Seidl, C. (1991): «Acceptance of Distributional Axioms: Experimental Findings», in W. Eichorn (ed), *Models and Measurement of Welfare and Inequality.*

Herrero, C. and Villar, A. (1989): «Comparaciones de renta real y evaluación del bienestar», *Revista de Economía Pública* 2 pp. 79-101.

Jenkins, S. and Lambert, P. (1993): «Ranking Income Distributions when Needs Differ», *Review of Income and Wealth* 39, 4, pp. 337-356.

Kakwani, N. C. (1984): «Welfare Ranking of Income Distributions», in *Advances in Econometrics* 3.

Kolm, S. C. (1976a): «Unequal Inequalities I», *Journal of Economic Theory* 12, pp. 416-442.

Kolm, S. C. (1976b): «Unequal Inequalities II», *Journal of Economic Theory* 13, pp. 82-111.

Kolm, S. C. (1972): *Justice et equité.* Paris: Editions du Centre National de la Recherche Scientifique.

Kondor, Y. (1975): «Value Judgements Implied by the Use of Various Measures of Income Inequality», *Review of Income and Wealth* 21, pp. 309-321.

Lewbel, A. (1989): «Household Equivalence Scales and Welfare Comparisons», *Journal of Public Economics* 39. pp. 377-391.

Moyes, P. (1987): «A New Concept of Lorenz Domination», *Economic Letters* 23, pp. 203-207.

Muellbauer, J. (1974a): «Inequality Measures, Prices, and Household Composition». *Review of Economic Studies* 41, pp. 493-504.

Muellbauer, J. (1974b): «Prices and Inequality: the United Kingdom Experience». *Economic Journal* 84, pp. 32-55.

Pollak, R. (1991): «Welfare Comparisons and Situation Comparisons», *Journal of Econometrics* 50, pp. 31-48.

Pollak, R. and T. Wales (1979): «Welfare Comparisons and Equivalent Scales» ,*American Economics Review, Papers and Proceedings* 69, pp. 216-221.

Roberts, F. (1980): «Price-Independent Welfare Prescriptions», *Journal of Public Economics* 13, pp. 277-297.

Ruiz-Castillo (1993): «La distribución del gasto en España de 1973-74 a 1980-81», in J. A, y L. Gutiérrez (ed), *Igualdad y distribución de la renta y la riqueza,* Madrid: Fundación Argentaria. There is an English version, «The Anatomy of Real and Money Inequality in Spain, 1973-74 to 1980-81», *Journal of Income Distribution,* forthcoming.

Ruiz-Castillo, J. (1994): «The Evolution of the Standard of Living in Spain, 1973-74 to 1980-81», Universidad Carlos III de Madrid, Working Paper, 94-10, Economic Series 04.

Sen, A. (1973): *On Economic Inequality*. Oxford: Clarendon Press.

Sen, A. (1976): «Real National Income», *Review of Economic Studies* 43, pp. 19-39.

Shorrocks, A. (1980): «The Class of Additively Decomposable Inequality Measurements», *Econometrica* 48, pp. 613-625.

Shorrocks, A. (1983): «Ranking Income Distributions», *Economica* 50, pp. 3-17.

Shorrocks, A. F. (1988): «Aggregation Issues in Inequality Measurement», in W. Eichhorn (ed), *Measurement in Economics*, New York: Springer-Verlag.

Tomás, J. M. and Villar, A. (1993): «La medición del bienestar mediante indicadores de "renta real": caracterización de un índice de bienestar tipo Theil», *Investigaciones Económicas* 17, pp. 165-173.

Zubiri, I. (1985): «Income Inequality as a Predictor of Welfare Inequality», SEEDS, Working Paper 40, Instituto de Economía Pública, Universidad del País Vasco.

Resumen

Aprovechando algunas de las lecciones que se extraen de la literatura sobre comparaciones intertemporales y/o espaciales de desigualdad de la renta, presentamos un marco de análisis completo para comparar el bienestar social o agregado de muestras independientes con información sobre la renta y otras características de los hogares. Este marco sirve para clarificar una variedad de asuntos ya tradicionales que versan sobre I) el dominio apropiado del problema de evaluación social; II) la necesidad de considerar nociones alternativas de desigualdad ante cambios en la media; III) la descomposición de los cambios en el bienestar real, entre cambios en la media a precios constantes y cambios en la desigualdad en términos reales; IV) la naturaleza de las comparaciones del bienestar de hogares heterogéneos que están implícitas en todo trabajo aplicado, y V) las fuertes consecuencias de los supuestos sobre separabilidad necesarios para la descomponibilidad de la desigualdad y el bienestar por subgrupos de la población.

Palabras clave: bienestar, desigualdad; escalas de equivalencia; descomponibilidad por subgrupos de la población.

Recepción del original, marzo de 1994
Versión final, diciembre de 1994