INVESTIGACIONES ECONOMICAS. Volumen XIX (2) - Mayo 1995, pp. 269-289

THE RELATIVE IMPORTANCE OF INFLATION AND CURRENCY DEPRECIATION IN THE DEMAND FOR MONEY: AN APPLICATION OF THE ESTIMATION BY SIMULATION METHOD TO THE GERMAN HYPERINFLATION

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The model presented in this paper assumes that the demand for money is a function of the expected inflation rate and expected depreciation rate of the domestic currency, characterizing the costs of holding money. In addition, the depreciation rate is assumed to follow a crawling peg rule in which it is adjusted in proportion to the gap between the rate of inflation and the rate of depreciation. The model is estimated using a GMM technique called estimation by simulation. The empirical results show that the model fits the data better than Cagan's model. Moreover, they show that foreign nominal assets were closer substitutes for domestic money than were real assets during the German hyperinflation.

1. Introduction

Identifying a stable demand for money during hyperinflationary periods was the main objective in Cagan's (1956) pioneering paper.[1] He assumes that the demand for money is only a function of the expected rate of inflation; that is, a function of the expected cost of holding money. His intuition was that during hyperinflationary periods the variation in the real variables determining the demand for money during regular periods (real income and real interest rate) is negligible compared to the variation in other relevant nominal variables. In this sense, the demand for money can be isolated from any real variable, and can be viewed only in terms of nominal variables; in particular, in

* This paper is a revised version of the second chapter of my Ph D. dissertation submitted to the Graduate School of State University of New York at Stony Brook in 1992. I thank Steve Cassou, two anonymous referees, and the editor of this Journal for their helpful comments. Of course, all errors are my own responsibility. Financial support from Ministerio de Educación (Spain) through project DGICYT PB92-0453 is gratefully acknowledged.

[1] Other papers trying to estimate a money demand function using data from hyperinflationary episodes are Sargent and Wallace (1973), Sargent (1977), Flood and Garber (1980), Burmeister and Wall (1987), Casella (1989) and Vázquez (1994).

terms of the anticipated rate of inflation. The simplicity of Cagan's model implies that a restrictive monetary policy reduces inflation quickly. The fact that this kind of policy has not been successful in past inflationary episodes may indicate that there are grounds for improving Cagan's model.

Abel, Dornbusch, Huizinga and Marcus (1979) investigate hyperinflation in a model which includes not only inflation but also currency depreciation as important determinants of demand for money. They show that the expected rate of inflation and the expected rate of depreciation of the domestic currency with respect to foreign currency should both be arguments in the money demand equation in order to reflect the costs of holding money during hyperinflationary periods.[2] This paper builds on the money demand model of Abel, Dornbusch, Huizinga and Marcus (1979), by assuming that the government controls the foreign exchange market mainly through manipulation of the exchange rate. Following Bruno (1989), the depreciation of the exchange rate follows a crawling peg rule in which the rate of depreciation is adjusted in proportion to the gap between the rate of wage inflation and the rate of depreciation. In addition, it is assumed that wage inflation is adjusted to changes in both the inflation rate and the depreciation rate.[3] These models consider the money supply as exogenous, trying to characterize in a simple setting how the consideration of the depreciation rate in the demand for money affects the path of the inflation rate when a restrictive monetary policy is implemented. Cagan's model implies that current inflation depends on the growth rates of money supply in the previous two periods. By contrast, the model presented in this paper implies that the current inflation rate depends on the immediate past inflation, on the current growth of money supply, and on the growth rate of money supply in the previous three periods. Therefore, the effect of a variation in the current growth of money supply on future inflation lasts longer in this model than in Cagan's model.

The model proposed in this paper and Cagan's model are estimated using a generalized method of moments (GMM) technique suggested by Lee and Ingram (1991), and Duffie and Singleton (1993), called estimation by simulation (ES). This estimation method was chosen for three main reasons. First, it allows a wide range of the model's features to be considered, such as simulating an unobserved variable instead of considering an alternative measure of it. Second, it is a non-instrumental variable GMM, which avoids the difficult issue of finding the appropriate instrumental variables when applying other GMM techniques. Finally, in contrast to the cointegration

[2] This characterization of the demand for money has been also used by Taylor (1991). His paper differs from this paper mainly in three aspects. First, the rational expectations hypothesis is not imposed. Second, no relationship between the inflation rate and the depreciation rate is established. Finally, his model is estimated by applying cointegration techniques.

[3] Both assumptions represent quite well some aspects of the scenarios in hyperinflationary economies.

technique used by Taylor (1991) to estimate the same demand for money, which requires the velocity shock of demand for money to be stationary, the estimation by simulation method allows the velocity shock to follow a random walk process. The random walk assumption is generally accepted in the literature (Sargent and Wallace (1973), Sargent (1977), Casella (1989), etc.) in order to capture the high persistence of velocity shocks during hyperinflation.

Maximum likelihood estimation (MLE) of the models studied in this paper is also feasible. The main advantage of using ES over MLE is that it allows to focus our attention on the ability of the models to explain observed empirical regularities characterized by the VAR's, whereas MLE fits the models globally. ES would present important advantages over MLE if the model presented in this paper were generalized to a non-linear model, considering temporal aggregation or unobservable shocks (see Duffie and Singleton (1993) for a wider discussion about the properties of the ES).

The data set used is the same data set considered by Abel, Dornbusch, Huizinga and Marcus (1979). The estimates obtained from the model proposed in this paper confirm the result obtained in Abel, Dornbusch, Huizinga and Marcus (1979), that foreign currencies were a closer substitute for domestic money than were real assets during the German hyperinflation of the 1920's. Moreover, this model provides a better fit to the data than Cagan's model. The empirical results also show that, in both models, the effect of a variation in the growth rates of the nominal money supply on future variations in inflation does not always have the same sign, although in Cagan's model this dynamic fluctuation is quantitatively less important than in this model. More precisely, in this model a one-period decrease in the current growth of the money supply decreases inflation today and tomorrow. However, inflation will increase and decrease alternately thereafter until the effect of this policy dies out. The same monetary policy in Cagan's model decreases inflation tomorrow, and also results in a small one-period increase in inflation after the first period. Consequently, the monetary authority has to be more patient in the model introduced in this paper than in Cagan's model. The intuition for this result is quite simple. The model proposed in this paper by assuming that the rate of depreciation is determined by past inflation rates implies a negative relationship between current inflation and past inflation rates because higher inflation rates in the past (higher depreciation rate in the current period) imply a lower inflation rate in the current period in order to clear the money market.

In order to check the robustness of these results we estimate the model proposed in this paper using Flood and Garber's (1980) data set. This data set has been frequently utilized in the literature (Burmeister and Wall (1987), Casella (1989), and Vázquez (1994)). The estimates obtained from the model suggested in this paper show large standard errors for some parameters. We detect an identification problem by analyzing the sensitivity of the estimation results to various choices of the set of statistics used in the estimation by simulation algorithm.

The rest of the paper is organized as follows. In Section 2, the model is described. In Section 3, the estimation by simulation technique is applied to estimate the models. Section 4 shows the estimation results. These results are also illustrated by a simulation analysis of the effects on the inflation rate path of a one-period restrictive monetary policy. Finally, Section 5 offers some concluding comments.

2. The model

The model assumes that foreign nominal assets and real assets are alternatives to domestic money. Following Abel *et al.* (1979), the demand for money has as arguments the expected rate of inflation and the expected rate of depreciation of the domestic currency with respect to foreign currencies. Formally, the money market equilibrium condition is written as:

$$m_t - p_t = \alpha_0 + \alpha\pi_t + \lambda d_t + u_t, \qquad [1]$$

$$u_t = u_{t-1} + v_t,$$

where m_t is the logarithm of the money supply, p_t is the logarithm of the price level, α_0 is a constant parameter, α and λ are negative parameters denoting the semi-elasticity of demand for money with respect to the inflation rate and depreciation rate, respectively; π_t is the expected inflation rate, defined by

$$\pi_t = E_t p_{t+1} - p_t, \qquad [2]$$

where E_t denotes the expectation operator conditional on the information available at time t; d_t is the expected depreciation rate, defined by

$$d_t = E_t e_{t+1} - e_t,$$

where e_t is the logarithm of the exchange rate; and v_t is a white noise error with mean zero and variance σ_v^2. According to equation [1] an increase in expected inflation or an increase in the expected rate of depreciation reduces the demand for real money balances.

Let us denote by Δe_t the rate of depreciation of the domestic currency with respect to a foreign currency, say the pound sterling. Formally, $\Delta e_t = e_{t+1} - e_t$, which implies that $d_t = E_t \Delta e_t$. During high-inflation periods the government usually controls the foreign exchange market in order to reduce pressure on the domestic currency. Following Bruno (1989), we assume that government intervention takes the form of a crawling peg rule in which the rate of depreciation is noisily adjusted in proportion to the gap between the rate of wage inflation and the rate of depreciation

$$\Delta e_t - \Delta e_{t-1} = \beta_1 (\omega_{t-1} - \Delta e_{t-1}) + \epsilon_t. \qquad [3]$$

where ω_t is the rate of wage inflation, and ϵ_t is a disturbance term with mean zero and variance σ_ϵ^2. As mentioned by Bruno (1989), the rationale for this adjustment rule comes from the supply of exports (or import substitutes) in the current account, that is, whenever the inflation wage departs from an equilibrium level, the loss (gain) in competitive power depreciates (appreciates) the domestic currency. Furthermore, Bruno (1989) also suggests that wage adjustments follow the rule:

$$\omega_t = \Phi \Delta p_t + (1 - \Phi) \Delta e_t, \quad [4]$$

where $\Delta p_t = p_{t+1} - p_t$ is the actual rate of inflation. This rule specifies wage adjustments as being a weighted average of domestic inflation and currency depreciation. The currency depreciation term is intended to capture the idea that workers adjust wages to higher import product prices. Another interpretation is that during high-inflation periods the rate of depreciation is an important component of expected inflation, so wages are adjusted to keep up with inflation. Combining equations [3] and [4], we obtain

$$\Delta e_t - \Delta e_{t-1} = \beta (\Delta p_{t-1} - \Delta e_{t-1}) + \epsilon_t, \quad [5]$$

where $\beta = \beta_1 \Phi > 0$. The first-order difference equation [5] can be written

$$\Delta e_t = \frac{\beta}{1 - (1-\beta)L} \Delta p_{t-1} + \frac{\epsilon_t}{1 - (1-\beta)L}. \quad [6]$$

Using the definitions of π_t and d_t and equation [6], equation [1] can be written as[4]

$$[1 - (1 - \beta) L] (m_t - p_t) = \alpha_0 \beta + \alpha [1 - (1 - \beta) L] (E_t p_{+1} - p_t) + + \lambda \beta (p_t - p_{t-1}) + \eta_t, \quad [7]$$

where $\eta_t = [1 - (1 - \beta) L] u_t + \lambda \epsilon_t$. The solution of this second-order difference equation, which is derived in Appendix 1, depends on the smaller characteristic root implicitly defined in [7], which we denote by μ_1. If $-1 < \mu_1 < 1$, the solution is represented by the following equation

$$p_t = -\frac{(\alpha_0\beta + \alpha\beta\gamma_0)}{A_0} + \frac{A_1}{A_0} p_{t-1} + A_2 m_t - (1-\beta) A_2 m_{t-1} + \frac{A_2}{\mu_2}[\sum_{i=0}^{\infty} (\mu_2)^{-i} E_t m_{t+i+1} - (1 - \beta) \sum_{i=0}^{\infty} (\mu_2)^{-i} E_{t-1} m_{t+i}] - (1 + \frac{\beta}{\mu_2 - 1}) (v_t + \beta u_{t-1}) - \lambda \epsilon_t, \quad [8]$$

where $A_0 = \alpha(\mu_1 - 1) + \lambda\beta + 1$, $A_1 = \alpha(1 - \beta)\mu_1 - \alpha(1 - \beta) + \lambda\beta + (1 - \beta)$, and $A_2 = \frac{1 - \alpha\delta}{A_0}$.

[4] Since Δ_{e_t} is a predetermined variable, $d_t = \Delta_{e_t}$

On the other hand, if $\mu_1 < -1$ the solution is represented by

$$p_t = \frac{\alpha_0\beta + \beta\alpha\delta_0}{B_0} + \frac{B_1}{B_0} p_{t-1} +$$

$$\frac{\alpha\delta_2}{\mu_2 B_0}[\sum_{i=0}^{\infty} (\mu_2)^{-i} E_t m_{t+i+1} - (1-\beta) \sum_{i=0}^{\infty} (\mu_2)^{-i} E_{t-1} m_{t+i}] -$$

$$\frac{\alpha\delta_1}{\mu_1 B_0}[\sum_{i=0}^{\infty} (\mu_1)^{-i} E_t m_{t+i+1} - (1-\beta) \sum_{i=0}^{\infty} (\mu_1)^{-i} E_{t-1} m_{t+i}] +$$

$$\frac{1}{B_0}[1 - \frac{\beta}{(\mu_2 - 1)(\mu_1 - 1)}][u_t - (1-\beta) u_{t-1}] + \frac{\lambda}{B_0} \epsilon_t, \qquad [9]$$

where $B_0 = \alpha - 1 - \lambda\beta$, and $B_1 = (1-\beta)\alpha - \lambda\beta - (1-\beta)$.

Note that in both solutions of the model, equations [8] and [9], the price level is a function of the immediate past level of price, that is, the model introduces some degree of inertia into the dynamics of the price level. Solutions [8] and [9] are bubble-free solutions. We focus our attention on bubble-free solutions for two reasons. First, several papers have tested the existence of bubbles during German hyperinflation but their results are rather inconclusive: Flood and Garber (1980), considering that money is exogenous, reject the existence of a bubble. Considering that money is endogenous, Burmeister and Wall (1987) are not able to reject the existence of a bubble. However, Casella (1989) does not reject the null of a bubble when money is considered exogenous, but the null is rejected when money is modeled as an endogenous variable. Second, although our money demand has not been explicitly derived from first principles of utility maximization, we do not want to introduce bubbles because they violate the transversality condition associated with agents' intertemporal optimization problems.

3. Estimating the model by simulation

The estimation method applied in this study, called estimation by simulation (ES), was suggested by Lee and Ingram (1991) and Duffie and Singleton (1993) to estimate models using time series data sets.[5] This method has been successfully applied in several contexts: Lee (1989) used ES to analyze the asset returns and inflation relationship, Vázquez (1994) to estimate a time-varying version of Cagan's model, Cassou (1993) to study stochastic dynamic

[5] MacFadden (1989) and Pakes and Pollard (1989) have proposed a similar simulation estimator for use in the discrete-response problem.

taxation, and Gardeazabal, Regúlez and Vázquez (1994) to test a generalized version of the canonical model of exchange rates.

As mentioned in the Introduction, ES has interesting properties which make the method appropriate in certain contexts. First of all, ES is an appropriate method for estimating non-linear macroeconomic models for which a close-form solution for the endogenous variables of the models cannot be obtained, especially in those models for which the moment conditions used by GMM cannot be found analytically in terms of the observable variables and the unknown parameters. Although our model is linear, ES still has some features which make it adequate for estimating this model. For instance, it allows us to estimate an unobservable variable such as the expected rate of inflation instead of considering an alternative measure of it. Moreover, it allows us to study important features of the model that cannot be obtained analytically from the model, such as VAR representations. Furthermore, the ES method is a non-instrumental variable GMM technique which avoids the always difficult election of the appropriate instrumental variables in order to implement other GMM methods. Finally, in sharp contrast to the cointegration method applied by Taylor (1991), which requires the velocity shock of the money demand equation to follow a stationary process, ES allows this velocity shock to follow a random walk (assumption generally accepted in the literature).

ES is a specific type of the generalized method of moments estimator which uses in the metric a set of statistics computed from the data set used and from a number of different simulated data sets generated by the model being estimated. In this study, the statistic used to implement the method are the VAR coefficients from a four lag, two variable system formed by the second differences of m_t and p_t.[6] To implement the method, we construct a $p \times 1$ vector with the VAR coefficients obtained from the real data, denoted by $H_T(\theta_0)$, where p in this application is 19,[7] T denotes the length of the time series data, and θ is a $k \times 1$ vector whose components are the underlying parameters of the model being estimated. The true parameter values are denoted by θ_0.

Given that the real data is by assumption a realization of a stochastic process, it is desirable to simulate the model a large number of times, say n. As a compromise, we make $n = 5$ in this application. For each simulation a $p \times 1$ vector of VAR coefficients, denoted by $H_{N_i}(\theta)$, is obtained from the time series of the second differences of m_t and p_t generated from the model being estimated, where $N = nT$ is the length of the simulated data. Averaging the n realizations of the simulated VAR coefficients, i.e, $H_N(\theta) = \frac{1}{n}\sum_{i=1}^{n} H_{N_i}(\theta)$

[6] To find the appropriate lag the likelihood ratio test was used. The null hypothesis tested was r lags versus $r + 1$ lags. The lowest number of lags r associated with the non rejection of the null is chosen.

[7] Since the constant terms of VAR coefficients measure the units we are not interested in these terms. We then have 16 coefficients from a four lag, two variable system, and three more coefficients from the non-redundant elements of the variance matrix of the residuals.

we obtain a measure of the expected value of the simulated VAR coefficients, $E(H_{N_i}(\theta))$.

The simulation estimator of θ_0 is obtained from the minimization of a distance function of VAR coefficients from real and simulated data. Formally,

$$\min_{\theta} J_T = [H_T(\theta_0) - H_N(\theta)]' \, W[H_T(\theta_0) - H_N(\theta)],$$

where W^{-1} is the covariance matrix of $H_T(\theta_0)$.

Denoting the solution of the minimization problem by $\hat{\theta}$, i.e, the simulation estimator, Lee and Ingram (1991) and Duffie and Singleton (1993) prove the following results:

$$\sqrt{T}(\hat{\theta} - \theta_0) \rightarrow N[0, (B'WB)^{-1}], \quad [10]$$

$$TJ_T \rightarrow \chi^2 (p - k), \quad [11]$$

where B is a full rank matrix given by $B = E\left(\frac{\partial H_{N_i}(\theta)}{\partial \theta}\right)$. For small values of n the variance of the estimated parameter vector is $(1+\frac{1}{n})\,(B'WB)^{-1}$; and the statistic in [11] should be $(1+\frac{1}{n})TJ_T$.

Giveng that the continuity of J_T is needed for the proof of the asymptotic normality of the ES estimator, as has been pointed out by Lee and Ingram (1991, p. 204), the implementation of the ES method requires that the random errors used to generate the simulated time series must be held fixed throughout the minimization algorithm because the objective function J_T is not continuous otherwise. Moreover, if we allow for a new random error draw for each iteration of the algorithm J_T changes even when the value of θ is held fixed. Therefore, in that case the minimization routine would have difficulty distinguishing a variation in J_T due to a new random error draw from a variation due to a change in θ.

Minimization of the objective function J_T was done using the optimization package MAXMUN programmed in GAUSS language. The algorithm BFGS was applied.[8] To calculate the covariance matrix we need to obtain B. Calculation of B requires two steps. First, obtaining the numerical first derivatives of the VAR's coefficients with respect to the estimates of the structural parameters θ for each of the n simulations. Second, averaging the n-numerical first derivatives to get B.

Since a sufficient condition for the simulation estimator to be consistent and asymptotically normal is that the time series used in the estimation should be covariance stationary, the model is transformed in the following way. First differences are taken in [8]

[8] BFGS refers to Broyden-Fletcher-Glodfarb-Shanno positive definite secant update algorithm.

$$\Delta p_{t-1} = \frac{A_1}{A_0} \Delta p_{t-2} + A_2 \Delta m_{t-1} - (1-\beta) A_2 \Delta m_{t-2} + \frac{A_2}{\mu_2}$$

$$[\sum_{i=0}^{\infty} (\mu_2)^{-i} (E_t m_{t+i+1} - E_{t-1} m_{t+i}) - (1-\beta) \sum_{i=0}^{\infty} (\mu_2)^{-i} (E_{t-1} m_{t+i} -$$

$$E_{t-2} m_{t+i-1})] - \frac{1}{A_0}$$

$$(1 + \frac{\beta}{\mu_2 - 1}) [v_t - (1-\beta) v_{t-1}] - \frac{\lambda}{A_0}(\epsilon_t - \epsilon_{t-1}), \qquad [12]$$

where $\Delta m_t = m_{t+1} - m_t$.

Second, following Evans (1978) and Casella (1989) we assume that the money supply process in German hyperinflation follows $AR(1)$ in the first difference of Δm_t. Formally,

$$\Delta m_t - \Delta m_{t-1} = c_0 + \rho (\Delta m_{t-1} - \Delta m_{t-2}) + z_t. \qquad [13]$$

Third, in Appendix 2 it is proved that for m_t defined by [13], the following expression holds

$$E_t m_{t+i+1} - E_{t-1} m_{t+i} = E_{t-1} \Delta m_{t+i} + a_i z_{t-1},$$

where $a_i = (i+1) + \rho a_{i-1}$, $a_0 = 1$. Then,

$$\sum_{i=0}^{\infty} (\mu_2)^{-i} (E_t m_{t+i+1} - E_{t-1} m_{t+i}) = \sum_{i=0}^{\infty} (\mu_2)^{-i} E_{t-1} \Delta m_{t+i} + D_0 z_{t-1}, \quad [14]$$

where $D_0 = \Sigma_{i=0}^{\infty} (\mu_2)^{-i} a_i$. Substituting [14] into [12], and making use of the Wiener-Kolmogorov formulae discussed in Hansen and Sargent (1980) and Sargent (1987),[9] we obtain close form expressions of the infinite sums in the LHS of [14],

$$\Delta p_{t-1} = \frac{A_1}{A_0} \Delta p_{t-2} + A_2 \Delta m_{t-1} - (1-\beta) A_2 \Delta m_{t-2} + \frac{A_2}{\mu_2}$$

$$[-\mu_2 \Delta m_{t-2} + \mu_2 (1 - \frac{1+\rho}{\mu_2} + \frac{\rho}{\mu_2^2})^{-1} (\Delta m_{t-2} - \frac{\rho}{\mu_2} \Delta m_{t-3}) - (1-\beta)$$

$$(-\mu_2 \Delta m_{t-3} + \mu_2 (1 - \frac{1+\rho}{\mu_2} + \frac{\rho}{\mu_2^2})^{-1} (\Delta m_{t-3} - \frac{\rho}{\mu_2} \Delta m_{t-4}))] +$$

[9] See Flood and Garber (1980) and Casella (1989) for other applications of the Wiener-Kolmogorov formulae in a similar context.

$$\frac{A_2 D_0}{\mu_2}[z_{t-1} - (1-\beta) z_{t-2}] - \frac{1}{A_0}(1 + \frac{\beta}{\mu_2 - 1})[v_t - (1-\beta)v_{t-1}] -$$

$$\frac{\lambda}{A_0}(\epsilon_t - \epsilon_{t-1}). \qquad [15]$$

Clearly, in this model the inflation rate is a function of past rates of money creation, but depends also on the immediate past rate of inflation.

Taking first differences in [15], and rearranging we have

$$\Delta p_t - \Delta p_{t-1} = \frac{A_1}{A_0}(\Delta p_{t-1} - \Delta p_{t-2}) + A_2\,((\Delta m_t - \Delta m_{t-1}) -$$

$$(2-\beta)(\Delta m_{t-1} - \Delta m_{t-2}) + (1 - \frac{1+\rho}{\mu_2} + \frac{\rho}{\mu_2^2})^{-1}[(\Delta m_{t-1} - \Delta m_{t-2}) -$$

$$\frac{\rho}{\mu_2}(\Delta m_{t-2} - \Delta m_{t-3})] + (1-\beta)[(\Delta m_{t-2} - \Delta m_{t-3}) - (1 - \frac{1+\rho}{\mu_2} + \frac{\rho}{\mu_2^2})^{-1}$$

$$((\Delta m_{t-2} - \Delta m_{t-3}) - \frac{\rho}{\mu_2}(\Delta m_{t-3} - \Delta m_{t-4}))] + \frac{D_0}{\mu_2}[(z_t - z_{t-1}) -$$

$$(1-\beta)(z_{t-1} - z_{t-2})])$$

$$-\frac{1}{A_0}(1 + \frac{\beta}{\mu_2 - 1})[(v_{t+1} - v_t) - (1-\beta)(v_t - v_{t-1})] - \frac{\lambda}{A_0}(\epsilon_{t+1} - 2\epsilon_t + \epsilon_{t-1}). \quad [16]$$

Similarly, the same steps can be followed to transform the model when $|\mu_1| > 1$, i.e, transforming [9]) we have

$$\Delta p_t - \Delta p_{t-1} = \frac{B_1}{B_0}(\Delta p_{t-1} - \Delta p_{t-2}) + \frac{\alpha\delta_2}{B_0}(-(\Delta m_t - \Delta m_{t-1}) +$$

$$(1 - \frac{1+\rho}{\mu_2} + \frac{\rho}{\mu_2^2})^{-1}[(\Delta m_{t-1} - \Delta m_{t-2}) - \frac{\rho}{\mu_2}(\Delta m_{t-2} - \Delta m_{t-3})] +$$

$$(1-\beta)[(\Delta m_{t-2} - \Delta m_{t-3}) - (1 - \frac{1+\rho}{\mu_2} + \frac{\rho}{\mu_2^2})^{-1}((\Delta m_{t-2} - \Delta m_{t-3}) -$$

$$\frac{\rho}{\mu_2}(\Delta m_{t-3} - \Delta m_{t-4}))]) - \frac{\alpha\delta_1}{B_0}(-(\Delta m_t - \Delta m_{t-1}) + (1 - \frac{1+\rho}{\mu_1} + \frac{\rho}{\mu_1^2})^{-1}$$

$$[(\Delta m_{t-1} - \Delta m_{t-2}) - \frac{\rho}{\mu_1}(\Delta m_{t-2} - \Delta m_{t-3})] + (1-\beta)[(\Delta m_{t-2} - \Delta m_{t-3}) -$$

$$(1 - \frac{1+\rho}{\mu_1} + \frac{\rho}{\mu_1^2})^{-1}((\Delta m_{t-2} - \Delta m_{t-3}) - \frac{\rho}{\mu_1}(\Delta m_{t-3} - \Delta m_{t-4}))]) +$$

$$\frac{\alpha}{B_0}(\frac{\delta_2 D_0}{\mu_2} - \frac{\delta_1 D_1}{\mu_1})[(z_t - z_{t-1}) - (1-\beta)(z_{t-1} - z_{t-2})]$$

$$\frac{1}{B_0}[1 - \frac{\beta}{(\mu_2 - 1)(\mu_1 - 1)}][(v_{t+1} - v_t) - (1-\beta)(v_t - v_{t-1})] +$$

$$\frac{\lambda}{B_0}(\epsilon_{t+1} - 2\epsilon_t + \epsilon_{t-1}), \qquad [17]$$

where $\delta_1 = \frac{\mu_1 - 1 + \beta}{\alpha(\mu_1 - \mu_2)}$, $\delta_2 = \frac{\mu_2 - 1 + \beta}{\alpha(\mu_1 - \mu_2)}$, and $D_1 = \Sigma_{i=0}^{\infty}(\mu_1)^{-i} a_i$.

Therefore, the model is represented by the price equation [16] when $|\mu_1| < 1$ (or by [17] when $|\mu_1| > 1$) and by the money supply equation [13].

Assuming [13] and Cagan's demand for money, that is,

$$m_t - p_t = \alpha_0 + \alpha\pi_t + u_t,$$

$$u_t = u_{t-1} + v_t,$$

it is easy to show that

$$\Delta p_t - \Delta p_{t-1} = \frac{1}{1-\alpha}((1-\gamma)[1 - (1-\rho)\gamma + \rho\gamma^2]^{-1}[(\Delta m_{t-1} - \Delta m_{t-2})$$

$$-\rho\gamma(\Delta m_{t-2} - \Delta m_{t-3})] + D_2 \Delta z_{t-1}) - \Delta v_{t-1}, \qquad [18]$$

where $\gamma = \frac{\alpha}{\alpha - 1}$ and $D_2 = \Sigma_{i=0}^{\infty}\gamma^i a_i$.

Comparing the reduced form equation for prices from Cagan's model [18] with the reduced form for the model presented in this paper, equation [16] (or [17]), we observe the following differences. First, the current change in the growth rate of money supply $(\Delta m_t - \Delta m_{t-1})$ does not affect current inflation in Cagan's model, but it does in the other model. Second, the immediate past variation in inflation, $(\Delta p_{t-1} - \Delta p_{t-2})$ affects the current change of inflation in the model suggested in this paper, while the same variable has no impact on the current variation of inflation in Cagan's model. Third, a variation in the growth rate of money supply affects future changes of inflation in a longer period of time in this model than in Cagan's model. The intuition for this dynamic behavior resides in the assumption that the depreciation rate is determined by past inflation rates as shown by equation [6]. Therefore, according to the money demand equation, equation [1], higher inflation rates in past periods result in a lower inflation rate in the current period.

The vector of parameters to estimate is $\theta_0 = (\alpha, \lambda, c_0, \rho, \sigma_v, \sigma_z, \sigma_\epsilon)$.[10] The simulated time series for $(\Delta p_t - \Delta p_{t-1})$ and $(\Delta m_t - \Delta m_{t-1})$ are constructed from the equations [16] (or [17]) and [13], respectively. The first three values of $(\Delta m_t - \Delta m_{t-1})$ and the third value of $(\Delta p_t - \Delta p_{t-1})$ from the sample are used as initial values in the simulation. The error terms v_t, z_t, and ϵ_t are simulated using a normal random number generator with standard deviation σ_v, σ_z, and σ_ϵ, respectively.

4. Empirical results

Two data sets are considered. First, Abel *et al.'s*, data set goes from January 1921 to June 1923. A monthly average of the consumer price index is employed. The money supply time series is obtained as the geometric average of the end of the month data for the money supply reported by two different sources (see Abel *et al.* (1979)). On the other hand, Flood and Garber's data set goes from January 1918 to June 1923. The wholesale price index is used, and both time series (money and prices) are measured at the middle of the month.

The estimation results using Abel *et al.'s* data set are reported in Table 1. The first column describes the estimates for the model introduced in this paper. It shows that the semi-elasticity of the demand for money with respect to real assets, α, and with respect to foreign nominal assets, λ, has the right sign. But α is not significantly different from zero for some confidence intervals. Moreover, the parameter λ is significantly greater than α, confirming the result of Abel *et al.* (1979) that foreign currencies were closer substitutes for domestic money than were real assets during German hyperinflation. On the other hand, contrary to other studies (Flood and Garber (1980), and Casella (1989)) that also assume an exogenous money supply, the parameters of the money supply are satisfactory estimated, that is, ρ and σ_z are significantly different from zero. The third column shows the parameter estimates when the model is simulated $n = 20$ times to evaluate J_T. Comparing the first and third columns we observe that the estimates of the significant parameters do not change significantly, but the time needed for estimation increases dramatically.[11]

[10] The parameter β was estimated separately by OLS in equation [5] because the solution of the model depends on β being greater or smaller than $\frac{2 - 4\alpha}{1 - 2(\lambda + \alpha)}$ (i.e., it depends on $|\mu_1|$ being greater or smaller than one). This avoids problems avoids problems in the estimation algorithm because the search of the estimated values can be made separately for both cases.

[11] This requires that the model be simulated fifteen times more and also that a synthetic time series four times larger be generated. The time needed to evaluate $J_T(\theta)$ for a given θ roughly multiplies by eight.

TABLE 1
Estimation results using Abel *et al.*'s data set

	Proposed model $n = 5$	Cagan's model $n = 5$	Proposed model $n = 20$
J_T	3.934491	7.392277	4.329852
α	−0.267968 (0.157119)	−1.503223 (0.328793)	−0.256207 (0.186701)
λ	−1.632870 (0.125579)	–	−1.618145 (0.190785)
c_0	−1.409748 (2.228572)	−0.028919 (0.137716)	−3.334365 (1.940453)
ρ	0.425128 (0.155213)	0.226550 (0.111221)	0.449338 (0.153440)
σ_v	0.001422 (0.345819)	0.108086 (0.003819)	0.000207 (1.116977)
σ_z	0.037837 (0.002594)	0.039516 (0.001599)	0.037732 (0.001483)
σ_ϵ	0.000315 (0.044181)	–	0.000156 (0.172123)
β	0.550889 (0.282622)	–	0.550889 (0.282622)
μ_1	−0.524482	–	−0.531265

Standard errors in brackets.

The equation [16] could be written using the estimates from Table 1 as

$$\Delta p_t - \Delta p_{t-1} = -0.5245\ (\Delta p_{t-1} - \Delta p_{t-2}) + 1.5890\ (\Delta m_t - \Delta m_{t-1}) +$$

$$1.0763\ (\Delta m_{t-1} - \Delta m_{t-2}) - 2.1292\ (\Delta m_{t-2} - \Delta m_{t-3}) +$$

$$0.2747\ (\Delta m_{t-3} - \Delta m_{t-4}) + \frac{A_2 D_0}{\mu_2}[(z_t - z_{t-1}) - (1 - \beta)(z_{t-1} - z_{t-2})] -$$

$$\frac{1}{A_0}(1 + \frac{\beta}{\mu_2 - 1})[(v_{t+1} - v_t) - (1 - \beta)(v_t - v_{t-1})] -$$

$$\frac{\lambda}{A_0}(\epsilon_{t+1} - 2\epsilon_t + \epsilon_{t-1}). \qquad [19]$$

Looking at this equation we observe that an increase in immediate past inflation decreases current inflation. This means that consecutive inflation rates are negatively related. The reason for this lies in the equilibrium equation [7]. To simplify the argument assume β equals one. Then, according to [7] the expected inflation is negatively related to past inflation. On the other hand, the sign of the effect of past variations of the growth rate of the nominal money supply on inflation is not homogeneous. We will see below that this result also appears in Cagan's model, although it is quantitatively less important. According to our estimates the models predict that following a restrictive monetary policy, inflation decreases in some periods and in other periods increases during the transitory path towards equilibrium.

The second column in Table 1 shows the estimates for Cagan's model, that is, joint estimation of equations [13] and [18]. In this model the estimate for α is −1.5032, higher than in the other model, and significantly different from zero. Using the estimates from Table 1 the equation [18] can be written as

$$\Delta p_t - \Delta p_{t-1} = 0.4624\,(\Delta m_{t-1} - \Delta m_{t-2}) - 0.0629\,(\Delta m_{t-2} - \Delta m_{t-3}) + \frac{D_2}{1-\alpha}\Delta z_{t-1} - \Delta v_{t-1}. \qquad [20]$$

Looking at this equation we could say that Cagan's model predicts the variation of the inflation rate almost perfectly through the immediate past variation in the growth rate of the nominal money supply, $\Delta m_{t-1} - \Delta m_{t-2}$, because the other variable, $\Delta m_{t-2} - \Delta m_{t-3}$, has a coefficient close to zero.

Comparing equations [19] and [20] we can observe that the effects of a monetary policy on the inflation path are less transparent in the model suggested in this paper than in Cagan's model. In particular, according to equation [19] a restrictive monetary policy today that reduces $(\Delta m_t - \Delta m_{t-1})$ for one period[12] implies reductions of the inflation rate in two consecutive periods (today and tomorrow), however, the inflation rate will increase and decrease alternately thereafter until the effect of this policy dies out. The same restrictive monetary policy in a scenario described by Cagan's model decreases the rate of inflation tomorrow, but practically leaves the rate of inflation constant afterwards. To illustrate the implications of this policy more precisely, assume $\Delta m_t - \Delta m_{t-1} = -0.1$, and $\Delta p_{t-1} - \Delta p_{t-2} = \Delta m_{t-1} - \Delta m_{t-2} = \Delta m_{t-2} - \Delta m_{t-3} = \Delta m_{t-3} - \Delta m_{t-4} = 0$. Figure 1 summarizes the simulated expected paths of inflation for both models as resulting from this one-period restrictive monetary policy. Clearly, this restrictive policy has an immediate effect on the model proposed in this paper, however, it has a one-period lag effect in Cagan's model. Moreover, the expected path of variation in inflation is more volatile in a scenario described by the model suggested in this paper than in a scenario described by Cagan's model.

From all this discussion about the implications of a restrictive monetary policy, we can draw the conclusion that the monetary authority needs to be

[12] This policy can be viewed as a shock in the money supply rule.

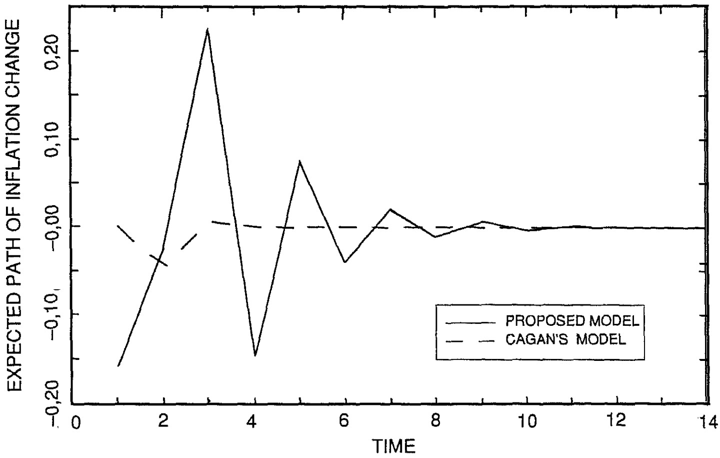

Figure 1
One period restrictive monetary policy effect on the expected path of inflation

more patient when implementing a restrictive monetary policy in our model than in Cagan's model.

We can test Cagan's model versus this model through the following statistic,

$$(1 + \frac{1}{n})\, T\, [J\,(C) - J\,(M)] \rightarrow \chi^2\,(3),$$

where n denotes the number of simulations done in the estimation by simulation algorithm, in this study $n = 5$; $T = 23$ denotes the length of the time series; $J\,(C)$ and $J\,(M)$ are the values of the objective function in the estimation by simulation algorithm for Cagan's model and the model introduced in this paper, respectively. This statistic takes the value 95.435. Since $\chi^2_{.05}\,(3) = 7{,}81$, we can conclude that the model presented in this paper fits the data better than Cagan's model.

Table 2 shows the estimation results using Flood and Gaber's data set.[13] The first column describes the estimates of the model introduced in this paper for the whole sample, and the second column shows the estimates for the subsample formed by the last thirty observations. At first sight, we observe that with Flood and Garber's data set the parameter λ has the wrong sign and furthermore α and λ display large standard errors. In order to study the possibility of an identification problem appropriately we follow Duffie and Singleton's (1993, p. 946) suggestions of analyzing, first, the sensitivity of the parameter estimates to alternative choices of lags for the VAR system. In this sense, we have tried to estimate the model by using a seven lag VAR system.

[13] For Flood and Garber's data set the statistics used in the estimation by simulation algorithm are the VAR coefficients from a six lag system.

TABLE 2
Estimation results using Flood and Garber's data set

	Proposed Mod. 55 Obs.	Proposed Mod. 30 Obs.
$\mathcal{J}_T$	2.260882	2.882709
α	−2.269284 (2.186796)	−1.800496 (1.593709)
λ	2.077595 (2.366132)	1.071464 (1.620392)
c_0	0.001316 (1.044727)	−0.026430 (0.772883)
ρ	−0.089316 (0.125296)	−0.051681 (0.157022)
σ_v	0.126772 (0.036426)	0.110437 (0.032792)
σ_z	0.052483 (0.001647)	0.068906 (0.002249)
σ_ϵ	0.041888 (0.128515)	0.105114 (0.115566)
β	1.159170 (0.162836)	1.159170 (0.162836)
μ_1	0.436407	0.239496

Standard errors in brackets.

However, the estimation by simulation algorithm does not converge (that is, a $\hat{\theta}$ which minimizes $\mathcal{J}_T$ is not found) even though the estimates of θ_0 obtained by using a six lag VAR system are utilized as initial values in the estimation by simulation algorithm. This result indicates that the parameter estimates are very sensitive to the choice of the lags included in the VAR system, meaning that the model is not well identified for this data set. Second, the rank of the B matrix (or equivalently the rank of the $B'B$ matrix) is analyzed.

TABLE 3
Eigenvalues for $B'B$ matrix

Flood-Garber 55 Obs.	Flood-Garber 30 Obs.	Abel *et al.*
0.003	0.015	0.003
0.010	0.025	0.649
0.039	0.167	1.337
0.084	0.606	4.992
1.061	0.980	21.736
3.184	8.385	358.309
17.371	31.504	905.813

The eigenvalues of the $B'B$ matrix for the estimates using different data sets are reported in Table 3. We observe that the eigenvalues using Flood and Garber's data set (first and second column) are relatively smaller compared with the eigenvalues obtained using Abel *et al.* data set (third column). This shows that the objective function J_T is relatively flat for some parameters when Flood and Garber's data set is used, which means that certain parameters are not well identified in this case.

5. Conclusions

In this paper we have introduced a hyperinflationary model which includes the rates of inflation and depreciation as arguments in the demand for money to reflect the costs of holding domestic money. In addition, it has been assumed that the rate of depreciation follows a crawling peg rule in which the rate of depreciation is noisily adjusted in proportion to the gap between the inflation rate and the depreciation rate. Analysis shows that Cagan's model implies that inflation is determined by the growth rates of the money supply in the previous two periods. However, the model presented in this paper implies that current inflation depends on immediate past inflation, on current growth of the money supply, and on the growth rates of the money supply in the previous three periods. Consequently, the effect of a reduction of the current growth of the money supply on inflation lasts longer in this model than in Cagan's model.

The models studied in this paper are estimated using a rather innovative time series estimation technique called estimation by simulation. This estimation method, designed to estimate time series models, was originally suggested by Lee and Ingram (1991). The empirical results obtained support the hypothesis that foreign currencies were closer substitutes for domestic money than were real assets during German hyperinflation when Abel *et al.'s* data set is employed. Furthermore, they show that the model introduced in this paper improves the fit in the data over Cagan's model. The empirical results also show that a one-period restrictive monetary policy in this model reduces inflation in two consecutive periods. However, inflation will increase and decrease alternately afterwards until the effect of this policy dies out. The same policy in Cagan's model decreases inflation tomorrow, and also results in a small one-period increase in inflation after the first period. Therefore, the monetary authority needs to be more patient when applying monetary policies in this model than in Cagan's model. The intuition for this result lies in the assumption that the rate of depreciation is determined by past inflation rates. Then, higher inflation rates in the past imply a lower inflation rate in the current period to guarantee the money market equilibrium. This negative relationship between current inflation and past inflation rates implied by the model introduced in this paper makes the associated inflation dynamics more complex than those obtained from Cagan's model.

The methodology applied in this paper can also be used to study other hyperinflationary episodes such as those in some South American countries

(Bolivia, Argentina, Peru and Brazil) in the 1980's and more recently in some Eastern European countries (for instance, Russia and Yugoslavia). However, on one hand those hyperinflationary episodes are much shorter than the *classic* German hyperinflation. On the other hand, those hyperinflations are sprinkled with unsuccessful economic reforms that make it more difficult to identify a stable demand for money during such episodes.

Finally, I should mention a drawback of this paper that is left for future research. The adjustment coefficient of the crawling peg rule for the rate of depreciation has been assumed to be constant. As pointed out by Bruno (1989), it could be plausible to assume that this coefficient is a function of the inflation rate, showing that the adjustment is faster when the inflation rate is higher.

Appendix 1

Taking expectations conditional on the information available at time *t*-1 in the equation [7], and rearranging, we obtain

$$\alpha E_{t-1}p_{t+1} + [\alpha(2-\beta)-\lambda\beta-1]\,E_{t-1}p_t - [(1-\beta)(1-\alpha)+(\lambda\,\beta)]\,E_{t-1}p_{t-1} = -\alpha_0\beta + E_{t-1}([1-(1-\beta)\,L]\,m_t - \eta_t). \qquad [A.1.1]$$

Applying the *B* operator to the last equation, we have

$$(B^{-2} - \varnothing_1 B^{-1} + \varnothing_2)\,E_{t-1}p_{t-1} = \gamma + \frac{1}{\alpha}\,E_{t-1}([1-(1-\beta)\,L]\,m_t - \eta_t), \qquad [A.\ 1.2]$$

where $\gamma = -\dfrac{\alpha_0\beta}{\alpha}$, $\varnothing_1 = \dfrac{\alpha(2-\beta)-\lambda\beta-1}{\alpha}$, and $\varnothing_2 = -\dfrac{(1-\beta)(1-\alpha)+\lambda\beta}{\alpha}$.

Writing the polynomial in equation [A.1.2] in terms of its characteristic roots,

$$(B^{-2} - \varnothing_1 B^{-1} + \varnothing_2) = (B^{-1} - \mu_1)(B^{-1} - \mu_2)$$

where $\varnothing_1 = \mu_1 + \mu_2$, and $\varnothing_2 = \mu_1 \mu_2$. Oscillatory solutions are rulet out.[14]

It is easy to show that $\mu_2 > 1$, and $\mu_1 < -1 \leftrightarrow \beta > \dfrac{2-4\alpha}{1-2(\lambda+\alpha)}$.[15]

[14] Complex solutions are ruled out if $\varnothing_1^2 - 4\varnothing_2 > 0$. Substituting the expressions of $\varnothing_1$ and $\varnothing_2$ in this inequality, and after some algebra we get the following inequality: $(1+\lambda\,\beta)^2 + \alpha^2\beta^2 - 2\alpha\beta + 2\alpha\lambda\beta^2 \geq 0$. This inequality is true since α and λ are negative, and β is non-negative by definition.

[15] To show that $\mu_2 > 1$, we substitute the definition of μ_2 in this inequality to get

$$\mu_2 = \frac{\varnothing_1 + \sqrt{\varnothing_1^2 - 4\,\varnothing_2}}{2} > 1,$$

after some algebra we see that the last inequality implies $\varnothing_1 - \varnothing_2 > 1$. Writing $\varnothing_1$ and $\varnothing_2$ in terms of the underlying parameters in the last inequality and operating we obtain $\beta > 0$. This inequality is true by definition. To show that $\mu_1 < -1 \leftrightarrow \beta > \dfrac{2-4\alpha}{1-2(\lambda+\alpha)}$, we use the definition of μ_1 to show that $\mu_1 = \dfrac{\varnothing_1 - \sqrt{\varnothing_1^2 - 4\,\varnothing_2}}{2} < -1$, implies $\varnothing_1 + \varnothing_2 < -1$. Using the definitions of $\varnothing_1$ and $\varnothing_2$ in the last inequality, after some algebra we obtain the above inequality.

Hence, if $-1 < \mu_1$, the general solution of the second-order difference equation [A.1.2] is

$$(B^{-1}-\mu_1) E_{t-1} p_{t-1} = \frac{\gamma}{1-\mu_2} + \frac{1}{B^{-1}-\mu_2} \frac{1}{\alpha} E_{t-1}([1-(1-\beta)L]\, m_t - \eta_t) + c\mu_2^t. \quad \text{[A.1.3]}$$

Since, $\mu_2 > 1$, we need to set $c = 0$ in order to get a stable solution. Rearranging the equation [A. 1.3], we have

$$E_{t-1} p_t = \gamma_0 + \mu_1 p_{t-1} - \frac{1}{\alpha} \sum_{i=1}^{\infty} (\mu_2)^{-i} E_{t-1}([1-(1-\beta)\, L]\, m_{t+i-1} - \eta_{t+i-1}),$$

where $\gamma_0 = \dfrac{\gamma}{1-\mu_2}$. After some algebra, the last equation could be written as

$$E_{t-1} p_t = \gamma_0 + \mu_1 p_{t-1} + \delta m_{t-1} - \frac{1}{\mu_2}\left(\frac{1}{\alpha} - \delta\right) \sum_{i=0}^{\infty} (\mu_2)^{-i} E_{t-1} m_{t+i} + W_t, \quad \text{[A.1.4]}$$

where $W_t = \dfrac{1}{\alpha} \sum_{i=0}^{\infty} (\mu_2)^{-i} E_{t-1}\eta_{t+i} = \dfrac{\beta}{\alpha(\mu_2-1)} u_{t-1}$, and $\delta = \dfrac{1-\beta}{\alpha\mu_2} < 0$,

whenever $\beta < 1$. Hence from [A.1.4] we have expressions for $E_{t-1} p_t$ and $E_t p_{t+1}$ that can be substituted into [7] to obtain equation [8].

If $|\mu_1| > 1$, the solution of the second-order difference equation [A.1.2] is

$$E_{t-1} p_t = \delta_0 + \frac{1}{\alpha(\mu_1-\mu_2)}\left[\frac{1}{1-(B\mu_2)^{-1}} - \frac{1}{1-(B\mu_1)^{-1}}\right] E_{t-1}[(1-(1-\beta)\, L)\, m_t - \eta_t], \quad \text{[A.1.5]}$$

where $\delta_0 = -\dfrac{\alpha_0 \beta}{\alpha(\mu_1-\mu_2)}\left(\dfrac{\mu_1}{1-\mu_1} - \dfrac{\mu_2}{1-\mu_2}\right)$. After some algebra, we obtain

$$E_{t-1} p_t = \delta_0 + \frac{\mu_2 - 1 + \beta}{\alpha(\mu_1-\mu_2)} \sum_{i=1}^{\infty} (\mu_2)^{-i} E_{t-1} m_{t+i-1} -$$

$$\frac{\mu_1 - 1 + \beta}{\alpha(\mu_1-\mu_2)} \sum_{i=1}^{\infty} (\mu_1)^{-i} E_{t-1} m_{t+i-1} + W'_t, \quad \text{[A.1.6]}$$

where $W'_t = -\dfrac{\beta}{\alpha(\mu_2-1)(\mu_1-1)} u_{t-1}$. As in the other case, we can now substitute $E_{t-1} p_t$ and $E_t p_{t+1}$ into [7] to obtain equation [9].

Appendix 2

In this appendix, it is shown that for m_t defined by

$$\Delta m_t - \Delta m_{t-1} = c_0 + \rho (\Delta m_{t-1} - \Delta m_{t-2}) + z_t. \qquad [A.2.1]$$

the following expression holds

$$E_t m_{t+i+1} - E_{t-1} m_{t+i} = E_{t-1} \Delta m_{t+i} + a_i z_{t-1},$$

where $a_i = (i+1) + \rho a_{i-1}$, $a_0 = 1$.

From [A.2.1] we obtain that

$$E_t m_{t+1} = c_0 + (2+\rho) m_t - (1+2\rho) m_{t-1} + \rho m_{t-2}. \qquad [A.2.2]$$

Hence,

$$E_t m_{t+1} - E_{t-1} m_t = (2+\rho) \Delta m_{t-1} - (1+2\rho) \Delta m_{t-2} + \rho \Delta m_{t-3}, [A.2.3]$$

On the other hand, using [A.2.1] we have that

$$E_{t-1} \Delta m_t = c_0 (2+\rho) + (1+\rho+\rho^2) \Delta m_{t-2} - \rho(1+\rho) \Delta m_{t-3}. \quad [A.2.4]$$

Operating in [A.2.3] we obtain

$$E_t m_{t+1} - E_{t-1} m_t = E_{t-1} \Delta m_t + (2+\rho) z_{t-1}.$$

Then, the result has been proved for $i = 0$. Following the same steps for $i = 1, 2, 3, \ldots$, we prove the result.

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Resumen

El modelo presentado en este artículo supone que la demanda de dinero es función de las tasas esperadas de inflación y de depreciación de la moneda nacional. Ambas variables caracterizan los costes de poseer dinero en lugar de otros activos. Además, se supone que la tasa de depreciación se ajusta en proporción a la diferencia existente entre la tasa de inflación y la tasa de depreciación. El modelo es estimado utilizando el método de estimación por simulación. Los resultados empíricos muestran que el modelo presentado se ajusta mejor a los datos que el modelo de Cagan. Además, también muestran que los activos nominales extranjeros fueron sustitutivos más cercanos a la moneda nacional que los activos reales durante la hiperinflación alemana.

Recepción del original, marzo de 1994
Versión final, mayo de 1995