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THE ANALYTICAL STRUCTURE OF SEQUENTIAL MODELS OF INNOVATION AND MARKET EVOLUTION

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Este artículo analiza la estructura de una serie de modelos recientes que estudian las relaciones existentes entre la introducción de una serie de innovaciones de forma secuencial, y la estructura de mercado de la industria que utiliza esas innovaciones. Para simplificar el tratamiento se presentan diversas versiones de un modelo con tres períodos y dos innovaciones en un contexto de duopolio. Particular atención se presta al examen de una reciente proposición de J. Vickers acerca de una supuesta relación entre los niveles de competencia o rivalidad entre el juego por las patentes y el juego de mercado del producto final.

1. Introduction

Since the early work of Schumpeter, the economic consideration of technological change has been related to the problem of market-structure dynamics. However, when this problem has been reconsidered in a formal way between 1962 and the early 80's, economists have concentrated their studies on the influence of market structure on the rate of technological change, and little attention has been paid to a central issue in Schumpeter's work: the impact of technological change on the evolution of market structure.

Gilbert and Newberry (1982), Kamien and Schwartz (1982) and Reinganum (1983) were the first ones in questioning the relative advantages and disadvantages of incumbent firms or new entrants in game-theoretical models of patent races. But in these models only games with just one innovation were considered and they did not explore the impact of a sequence of innovations on the initial market structure.

In Reinganum (1985)[1], this problem was addressed for the first time, but the structure of the model is extremely simplified, as Reinganum considers only drastic innovations that preserve the initial monopolistic structure.

The class of models that seems to have established itself as «the» appropriate way of approaching and tackling this problem has its roots in Vickers (1986).

[1] The excellent paper by M. T. Flaherty (1980) which presents a very different approach to that in the papers discussed here, merits separate mention.

In this paper, Vickers studies conditions for the persistence of technological leadership by a firm (what he calls «increasing dominance») and for variations in the identity of the firm with lowest marginal costs («action-reaction» in Vickers terms). Beath *et al.* (1987) adapt Vickers' model of process innovations to a setting with product differentation and new product innovations, and Delbono (1988) uses a similar structure to incorporate in the model gradual approximations in the technological level of both firms, although he incorporates important differences in the auctioning process [2].

In this «family» of models, a sequence of new technologies is produced, and two duopolists that compete in a final-product market bid in an auction for the exclusive right to use a patent. In this auction, there is no strategic conduct whatsoever on the side of the two duopolists, and both players have perfect information, not only about the characteristics of the new technology, but also about their rival. Furthermore, there is no uncertainty about getting the patent and if a firm outbids its rival in the auction, it will win the patent with certainty. In this sense, the game is deterministic, as Delbono admits, and given the parametric data of the game (market conditions and initial cost structure) the final equilibrium is known by each player before the game starts.

In spite of this lack of strategic behaviour in the technology game, these authors establish some links between the type of duopolistic competition in the final-product market and the evolution of the market structure. Leaving aside some qualifications made by Beath *et al.*, all these authors point out that low (high) levels of competition in the final-product market imply leapfrogging (increasing dominance) in the technological leadership of the market.

All the models of sequential innovation, including Reinganum's, concur in predicting quite simple (sufficient) conditions for the occurrence of permutations in technological leadership. In the real world, we find lots of cases of technological asymmetry between firms competing in national or international markets that are perpetuated over time. Changes in technological leadership are the exception and not the rule. However, it seems hazardous to impute this generality to the existence of high levels of rivalry in the final-product markets. This discrepancy between the predictions provided by economic theory and reality, is our motivation to examine closely the structure of this family of models, because this could help us to learn to what extent these predictions are induced by the particular way in which the models have been set up.

This is the main concern of this paper. We think that the predictions of this family of models are highly dependent on the structure of the game, which has many possible equilibria besides increasing dominance and action reaction in technological leadership. For that purpose we will proceed in two sta-

[2] Harris and Vickers (1987) provides a completely different approach to study the influence of technological change on market structure, but, to our knowledge, this line of research has not been developed any further.

ges. First, we will study the characteristics of different equilibria in this family of models using a simplifed version of the original one proposed by Vickers. On a second stage, we will modify the structure of the game to study how these modifications alter the results.

One important feature of these models is that they relate the outcome of the technology game with the market conditions prevailing in the final-good market. Identifying the nature of those relationships will be the purpose of section V. We examine Vickers' results from a comparative-statics point of view. In this way, we can get a better idea about the generality of his results.

In the next sections, II to IV, we present a version of Vickers' model simplified to only two periods, and an initial one in which there is no innovation. Despite its simplicity, this model captures all the features of the models used by Vickers and Beath *et al.* But this simplification helps us to identify the direct benefits that stem from winning the patent, and those derived from the strategic advantage in future auctions that winning today implies. Furthermore, with this simple game structure, we can find necessary and sufficient conditions for four different types of equilibria which are combinations of the two simple types of equilibria studied by these authors.

The last sections of the paper present several modified versions of the basic model. As the results of these models depend heavily on the structure of the game, we think that, only by adapting the model to particular cases, we can draw conclusions that are applicable to those cases.

2. The model and preliminary results

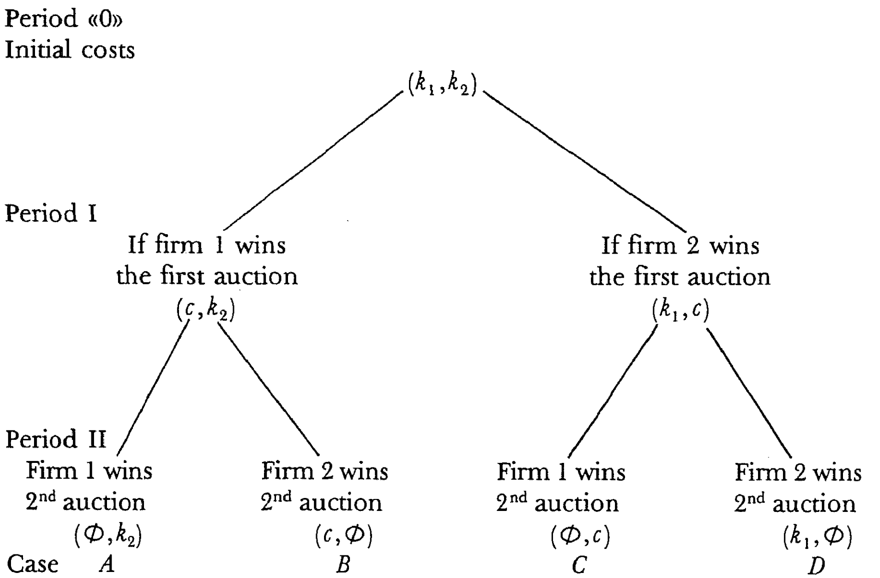
Our model consists of two different but related games. First, we will assume that two firms producing a perfect substitute compete as duopolists in a final-good market. On the other hand, a technology game is played by these two firms. Both firms are competitors to gain cost-reducing patents to produce the final good[3].

Initially, at time zero, firm 1 has lower constant average costs than firm 2, i. e. $k_1 < k_2$. At the end of this period, two new process innovations are announced and both firms bid for the new patents. The winner of the first patent will be able to produce the final good with average costs c during time period I. The winner of the second patent will enjoy average costs $\Phi < c < k_1 < k_2$ during period II. The firm with the highest reservation price for each patent will be the winner of the auction. However the winner will not have to pay his reservation price to get the patent but just a small amount above the bid made by his rival, i.e. his rival's reservation price. Finally, we shall assume that the cost reductions brought about by the innovations will not be drastic enough to eliminate the duopolistic market structure.

[3] As Delbono points out, this kind of games are quite general as they can be adapted to study product innovations.

The nature of the game and the resulting cost structure in each period are shown in game 1.

Game 1



Note that this collection of cases includes the two cases studied by Vickers: «*A*» would coincide with what he calls «increasing dominance» and «*C*» is a three period example of «action-reaction»[4]. However, we include two extra possibilities which are but combinations of the other two. Thus, we allow for one firm «overtaking» the other but in two periods instead of only one, and one firm passing its rival and building an increasing dominance afterwards.

Nevertheless, we still keep Vickers' «non-historical» approach in the sense that one firm can bid for a patent under the same conditions than his rival can, irrespective of the difference in their initial cost levels.

We look for a perfect equilibrium, in the sense that we will require time consistency. The two innovations will be announced at the end of period «0». Both firms will make their calculations about their own and their rival's reservation prices and each one of them will know who will win each patent. Hence, reservation prices for each patent will reflect not only present conditions, i.e. current costs for each firm, but also:

[4] According to Vickers, there will be «increasing dominance» when the firm that enjoys an initial cost advantage maintains and increases that advantage by winning all the subsequent new patents, while its rival keeps its initial cost level all along the duration of the game. On the contrary, the term «action-reaction» denotes a kind of equilibrium in which the identity of the low-cost firm alternates from one period to the next one. As a certain strand of the literature has kept this nomenclature (see Beath *et a.* (1987) and DelBono (1988)), we shall adopt it here too.

- the «past», because current costs will depend on who won the last patent, and
- the «future», because the identity of the winner of today's auction will have an impact on the future auction. This impact will be twofold. First, whoever wins the second patent will have to face a competitor with a given cost level, and that level will be determined by the outcome of the previous auction. Secondly, the cost levels with which firms face the second auction will have an influence on their respective chances of winning the second auction, but those initial cost levels will be the result of the first auction [5].

In the rest of this section, we shall try to find necessary and sufficient conditions for each type of equilibrium. As it is usual in this kind of models, we shall proceed backwards. For simplicity we shall leave aside the effect of discounting.

2.1. *Period II*

As this is the final period, the value of the patent for any firm will be just the difference in current profits for that firm in the only two possible events: when it wins the second patent and when it loses the auction. Each firm will have two different reservation prices depending on who won the first auction.

In cases A and B, i.e. when firm 1, the initially low cost firm wins the first patent, the reservation prices for the second patent for firms 1 and 2 will be,

$$P^1_{II} = \pi^1_{II}(\Phi,k_2) - \pi^1_{II}(c,\Phi) \text{ and} \quad [1]$$

$$P^2_{II} = \pi^2_{II}(c,\Phi) - \pi^2_{II}(\Phi,k_2) \quad [2]$$

where P_{it} and π_{it} denotes reservation prices and profits for firm $i = 1,2$ in period $t = \text{I,II}$. The first (second) argument in the profit function indicates the cost level for firm 1 (2).

In cases C and D, i.e. when firm 2, the high cost firm at the beginning of the game, wins the first patent, reservation prices will be,

$$P^1_{II} = \pi^1_{II}(\Phi,c) - \pi^1_{II}(k_1,\Phi) \text{ and} \quad [3]$$

$$P^2_{II} = \pi^2_{II}(k_1,\Phi) - \pi^2_{II}(\Phi,c). \quad [4]$$

[5] Alternatively, one could think of a «closed-loop» perfect equilibrium, that would be time consistent forwards too. We have worked out this model, which is not exactly the same, but the main conclusions are basically the same. However, this departure from Vickers's original model has the advantage of allowing us to compute the absolute and relative impact of the existence of a second patent race on the firm's reservation prices for the first innovation. In that case, the existence of a second patent raises the reservation price for the first patent in both firms, but this increment is relatively bigger in the initially high cost firm. This asymmetry of the model will be commented later.

Comparing two by two these equations we can know which firm wins the second patent. However, to compute the net payoffs for each firm, we will have to subtract the price of the patent from the profits that it will make in the second period. This will give us the following payoffs, V, for each firm in each case,

a) CASE A:

firm 1: $$V^1_{II} = \pi^1_{II}(\Phi,k_2) - \pi^2_{II}(c,\Phi) + \pi^2_{II}(\Phi,k_2) = \\ = \sigma_{II}(\Phi,k_2) - \pi^2_{II}(c,\Phi)$$

firm 2: $$V^2_{II} = \pi^2_{II}(\Phi,k_2)$$

b) CASE B:

firm 1: $$V^1_{II} = \pi^1_{II}(c,\Phi)$$

firm 2: $$V^2_{II} = \pi^2_{II}(c,\Phi) - \pi^1_{II}(\Phi,k_2) + \pi^1_{II}(c,\Phi) = \\ = \sigma_{II}(c,\Phi) - \pi^1_{II}(\Phi,k_2)$$

c) CASE C:

firm 1: $$\pi^1_{II} = \pi^1_{II}(\Phi,c) - \pi^2_{II}(k_1,\Phi) + \pi^2_{II}(\Phi,c) = \\ = \sigma_{II}(\Phi,c) - \pi^2_{II}(k_1,\Phi)$$

firm 2: $$V^2_{II} = \pi^2_{II}(\Phi,c)$$

d) CASE D:

firm 1: $$V^1_{II} = \pi^1_{II}(k_1,\Phi)$$

firm 2: $$V^2_{II} = \pi^2_{II}(k_1,\Phi) - \pi^1_{II}(\Phi,c) + \pi^1_{II}(k_1,\Phi) = \\ = \sigma_{II}(k_1,\Phi) - \pi^1_{II}(\Phi,c)$$

where σ_{II} are the industry joint profits in period two.

2.2. *Period I*

As we pointed out before, reservation prices for the first patent will capture the influence of the result of the first auction on the second one. For that purpose, we shall define the value functions, for each firm, of winning and losing the first auction.

Let $\mu_i(r,s)$ be the present value of all present and future net auction bids for firm $i = 1, 2$, when firm 1 has costs r and firm 2 has costs s. Hence, if firm 1 wins the bid for the first patent, its value function will be,

$$\mu_1(c,k_2) = \pi^1_I(c,k_2) + \max\{\pi^1_{II}(\Phi,k_2) + \pi^2_{II}(\Phi,k_2) - \\ \pi^2_{II}(c,\Phi), \pi^1_{II}(c,\Phi)\}$$

or,

$$\mu_1(c,k_2) = \pi^1_I(c,k_2) + \max\{\sigma_{II}(\Phi,k_2) - \pi^2_{II}(c,\Phi), \pi^1_{II}(c,\Phi)\} \quad [5]$$

while, if firm 1 loses the bid for the first auction, we will have,

$$\mu_1(k_1,c) = \pi_{I}^{1}(k_1,c) + \max\{\pi_{II}^{1}(\Phi,c) + \pi_{II}^{2}(\Phi,c) - \pi_{II}^{2}(k_1,\Phi),\ \pi_{II}^{1}(k_1,\Phi)\}$$

or,

$$\mu_1(k_1,c) = \pi_{I}^{1}(k_1,c) + \max\{\sigma_{II}(\Phi,c) - \pi_{II}^{2}(k_1,\Phi),\ \pi_{II}^{1}(k_1,\Phi)\} \quad [6]$$

As firm's 1 reservation price for the first patent is just the difference between the maximum profits that this firm will make if it wins the auction and the profits resulting from losing the auction, this price, P_{I}^{1} will be

$$P_{I}^{1} = \mu_1(c,k_2) - \mu_1(k_1,c) \quad [7]$$

Similarly, we could define this reservation price for firm 2 as,

$$P_{I}^{2} = \mu_2(k_1,c) - \mu_2(c,k_2) \quad [8]$$

where

$$\mu_2(k_1,c) = \pi_{I}^{2}(k_1,c) + \max\{\sigma_{II}(k_1,\Phi) - \pi_{II}^{1}(\Phi,c),\ \pi_{II}^{2}(\Phi,c)\} \quad [9]$$

and

$$\mu_2(c,k_2) = \pi_{I}^{2}(c,k_2) + \max\{\sigma_{II}(c,\Phi) - \pi_{II}^{1}(\Phi,k_2),\ \pi_{II}^{2}(\Phi,k_2)\} \quad [10]$$

2.3. *Definition of equilibria*

From the definitions above, we can conclude that the necessary and sufficient conditions to have «increasing dominance» (case A), as the equilibrium in our game, will be those that enable us to have at the same time

$$P_{I}^{1} > P_{I}^{2} \text{ and } P_{II}^{1} > P_{II}^{2}$$

and the equilibrium will bring «action-reaction» (case C) if and only if

$$P_{I}^{1} < P_{I}^{2} \text{ and } P_{II}^{1} > P_{II}^{2}$$

Finally, for the other two equilibria we have,

$$P_{I}^{1} > P_{I}^{2} \text{ and } P_{II}^{1} < P_{II}^{2} \text{ in case } B \text{ and}$$

$$P_{I}^{1} < P_{I}^{2} \text{ and } P_{II}^{1} < P_{II}^{2} \text{ in case } D.$$

Before we proceed to find the equilibrium conditions and discuss them, we think that it will be convenient for those purposes to comment very briefly some of the main characteristics of our game. Among the most important ones we will mention the following:

1. The game is finite, and the cost structure after the last auction will have an impact on the last period current profits only. If we assume that both firms

keep on competing in the product market after the technology game has finished, the reservation price for the last patent will be higher for both firms, and this will have induced effects on the other previous auctions.

2. As patents are allocated in auctions, only the winner has to incur in costs in the patent race. In this way, we lose one of the most important features of R&D races. As Dasgupta (1988) and other have pointed out, in patent races, losers do not get any return to their efforts and winners get all the rewards. Here, there is no cost derived from losing an auction other than operating with higher costs in the product market. Hence, we are not taking into account an important incentive to win the patent. A priori, there is no reason to believe that this incentive will be different for each firm, (unless we add to the initial cost asymmetry between firms, some kind of asymmetry in the R&D costs that each firm has to incur in order to achieve the same innovation).
3. The auctioning process has some peculiarities that do have a strong influence on the outcome of the game, as we will see in the next section. Who wins the patent is determined by the difference in reservation prices, but the cost of the patent is equal to the bid made by the firm that loses the auction. This implies that wining todays auction has an impact on future auctions. This influence is twofold: first, it determines who will be the low cost firm in the next auction, and second, it has an indirect effect on the reservation prices for the next patent, and hence, on the possibilities of winning that next patent and on the payoffs derived from doing so.

With these considerations in mind, we proceed to characterize each kind of equilibrium.

3. Necessary and sufficient conditions for each kind of equilibrium

In the game described in the former section, the identity of the winner of the *second* auction will depend just on the sign of the following two expressions,

$$\sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) \text{ and} \quad [11]$$

$$\sigma_{II}(k_1,\Phi) - \sigma_{II}(\Phi,c) \quad [12]$$

If firm 1 is the low-cost firm and [11] is positive (negative), firm 1 (2) will win the second and final patent. But if firm 2 is the low cost firm and [12] is positive (negative) firm 2 (1) will be the winner of the second patent.

The identity of the winner of the first auction depends on the sign of $(P_I^1 - P_I^2)$. But note that the «max» expressions in [5], [6], [9] and [10] will depend precisely on the sign of [11] and [12]. This relationship reduces the possible specifications of differences between P_I^2 and P_I^2 as defined in [7] and [8] to only four possible cases. These are expressed by equations [13] – [16] that summarise the values of the differences in reservation prices for first patent that the two companies will have depending on the outcome of the second auction:

$$P_I^1 - P_I^2 = \sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(\Phi,k_2) - \sigma_{II}(\Phi,c) + \pi_{II}^2(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) - \sigma_{II}(\Phi,c) \quad [13]$$

$$P_I^1 - P_I^2 = \sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(\Phi,k_2) - \sigma_{II}(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) - \pi_{II}^1(k_1,c) \quad [14]$$

$$P_I^1 - P_I^2 = \sigma_I(c,k_2) - \sigma_I(k_1,c) + \pi_{II}^2(k_1,\Phi) - \pi_{II}^1(\Phi,k_2) \quad [15]$$

$$P_I^1 - P_I^2 = \sigma_I(c,k_2) - \sigma_I(k_1,c) + 2\sigma_{II}(c,\Phi) - \pi_{II}^2(k_1,\Phi) - \sigma_{II}(k_1,\Phi) - \pi_{II}^2(\Phi,k_2) \quad [16]$$

Following the process of elimination that can be found in the appendix, [11] and [12] together with [13] – [16], we can get these necessary and sufficient conditions for each type of equilibrium,

Case «A» will occur if and only if:

$A1$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) > 0 \;;\; \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) > 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(\Phi,k_2) - 2\sigma_{II}(c,\Phi) + \pi_{II}^2(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) > 0$$

or if

$A2$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) > 0 \;;\; \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) < 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(k_2,\Phi) - \sigma_{II}(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) - \pi_{II}^1(k_1,\Phi) > 0$$

Case «B» will occur if and only if:

$B1$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) < 0 \;;\; \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) < 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + 2\sigma_{II}(\Phi,c) - \sigma_{II}(\Phi,k_1) - \pi_{II}^1(k_1,\Phi) - \pi_{II}^2(\Phi,k_2) > 0$$

or,

$B2$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) < 0 \;;\; \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) > 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + \pi_{II}^2(k_1,\Phi) - \pi_{II}^1(\Phi,k_2) > 0$$

Case «C» will occur if and only if:

$C1$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) < 0 \;;\; \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) > 0$ and

$$\sigma_{II}(\Phi,k_2) - \sigma_{II}(\Phi,k_1) + \pi_{II}^2(k_1,\Phi) - \pi_{II}^2(\Phi,k_2) < 0$$

or,

$C2$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) > 0 \;;\; \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) > 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(\Phi,k_2) - 2\sigma_{II}(c,\Phi) + \pi_{II}^2(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) < 0$$

Case «D» will occur if and only if:

$D1$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) > 0 \quad ; \quad \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) < 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(k_2,\Phi) - \sigma_{II}(k_1,\Phi) + + \pi^2_{II}(\Phi,k_2) - \pi^1_{II}(k_1,\Phi) < 0$$

or,

$D2$ $\quad \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) < 0 \quad ; \quad \sigma_{II}(c,\Phi) - \sigma_{II}(\Phi,k_1) < 0$ and

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) + 2\sigma_{II}(\Phi,c) - \sigma_{II}(\Phi,k_1) - \pi^1_{II}(k_1,\Phi) - \pi^2_{II}(\Phi,k_2) < 0$$

4. Discussion of equilibria

The necessary and sufficient conditions presented above are a generalization of the results in Vickers (1986). In his paper, Vickers concludes that the following assumption is sufficient to have action-reaction in an n-period model:

Assumption 1 (Vickers). For any period t and cost levels $k > 0$, $c > 0$ such that $k > c$,

$$\sigma_t(c,k+\varepsilon) < \sigma_t(c,k)$$

i.e. industry joint profits will be higher the lower the cost level of the high-cost firm is.

However, the inverse of assumption 1, (i.e., that joint profits are increasing with respect to the cost of the highcost firm), is not sufficient to ensure increasing dominance. For this later case, Vickers only provides a necessary condition, namely, that the high cost firm is making zero profits.

As assumption 1 is fulfilled in the Cournot-Nash case with linear demand function, Vickers goes on to identify action-reaction in the technology game with low levels of rivalry in the market for the final good that both firms produce. On the contrary, in his second proposition, Vickers finds that zero profits in the high cost firm is a necessary condition for increasing dominance[6]. From there, Vickers concludes that a high level of rivalry in the output market is a necessary condition for a stable condition of increasing dominance in the technology game.

Before considering those conclusions, we will give an economic interpretation to our results. In a recent paper, Beath *et al.* (1989) point out the two kinds of incentives that oligopolistic firms with different costs have in patent races. First, there is the incentive provided by the cost reduction that the patent will produce. If two firms have different average production costs, they will have different incentives to engage in a cost reducing investment. This incentive is independent from the fact that firms are competing in a patent race, and it can

[6] This is consistent with our sufficient conditions $A1$ and $A2$.

be computed as the difference in profits made by the firm with and without the patent. But on the other hand, oligopolistic firms will also compete for a patent because there is a competitive threat in patent races: losing a patent race implies that a rival has won it, and hence, you will face a stronger competition in the product market, in other words, there are benefits derived from being the first to innovate. The magnitude of this incentive can be known only in an indirect way.

In any case, in the race for just one patent, without any further future innovations (as it happens with the second patent in our model), the sum of these two effects for each firm is reflected by reservation prices such as those given in [1] and [2], and we will call it *direct effect.* As we saw before, which firm will have the highest total incentive or reservation price will depend on the sign of expressions [11] or [12]. This is clear because the profit incentive can be measured as,

$$\Omega^1 = \pi^1(\Phi,k_2) - \pi^1(c,k_2) \text{ and } \Omega^2 = \pi^2(c,\Phi) - \pi^2(c,k_2)$$

and the total effect is given by [1] and [2]. The incentive derived from the competitive threat will be the difference between,

$$P^1 - \Omega^1 = \Phi^1 = \pi^1(c,k_2) - \pi^1(c,\Phi)$$
$$P^2 - \Omega^2 = \Phi^2 = \pi^2(c,k_2) - \pi^2(\Phi,k_2)$$

Consequently, the differences in profit incentives and incentives due to the competitive threat can be defined as:

$$\Omega^1 - \Omega^2 = [\pi^1(\Phi,k_2) - \pi^1(\Phi,c)] + [\pi^1(k_2,c) - \pi^2(k_2,c)] \qquad [17]$$

$$\Phi^1 - \Phi^2 = [\pi^1(k_2,\Phi) - \pi^1(c,\Phi)] + [\pi^2(k_2,c) - \pi^1(k_2,c)] \qquad [18]$$

Note that the last two terms in brackets in [17] and [18] have the same absolute value but different sign, and therefore, they cancel out and the combined affect of the profit incentive and the competitive threat is just the result of the addition of the first two terms. Notice that this combined effect is equal to [11] and consequently, its sign will be the same as that of [11]. However, which incentive will be greater in absolute value for each firm depends not just on the sign of [11] but also on the relative difference in profits between the two firms prior to the auction, i.e. the second term in brackets.

As a matter of fact, the combined direct effect can be positive or negative, irrespective of the sign of [17] and [18]. However, which firm will have a higher incentive to get the patent coming from the profit incentive or from the competitive threat, will depend on the initial cost difference between k_2 and c. Actually, for a given value of the terms in [11], if the initial cost difference is relatively small, the profit incentive will be greater for the low cost firm, and the competitive threat will be greater (smaller) for the high cost firm, but which one will be dominant will depend just on the sign of [11]. This is so because the first term in brackets in [17] will always be positive and the first term in brackets in [18] will always be negative, but [17] will turn negative only if the cost asymmetry between c and k_2 is big enough to make $[\pi^1(k_2 - c) - \pi^2(k_2 - c)]$ negative and big enough, and [18] will turn positive only if $[\pi^2(k_2 - c) - \pi^1(k_2 - c)]$ is positive and big enough. On the other hand, if the initial cost difference is relatively large, the profit incentive will be greater for the high cost firm, and the incentive derived from the competitive threat will be greater for the low cost firm, but again, the net effect will be given by the sign of [11].

In a model with a sequence of innovations and patent races, the combination of these two effects still remains and can be found in the terms $\sigma(c,k_2) - \sigma(c,k_1)$ in [13]–[16]. However, when a sequence of innovations is considered, the former two incentives are not the only ones. In the particular auctioning system considered here (as well as in Vickers (1986) and others), the following effects, that we will call induced effect, are introduced in the n-1 first auctions:

1.º If a given firm wins today's auction, that firm will become the low cost firm in the next period, and consequently, that firm will have lower or higher incentives to win the next auction, depending on the sign of [11] and [12]. That will have a negative or positive influence of the willingness to pay for the patent currently being auctioned Hence, the direct effect that arises from the profit incentive and from the competitive threat can be reinforced or offset by the existence of future auctions.

2.º If a given firm wins today's auction and [11] and [12] are negative, his competitor will have a higher direct net incentive to win tomorrow's race. Consequently, to win tomorrow's patent, our firm will have to pay a higher price for it, as his rival's reservation price will be higher. Again, this effect will encourage or discourage firms to win today's patent depending on the sign of [11].

Our necessary and sufficient conditions above reflect the combinations of these effects in the two auctions. Notice first, that some necessary conditions to have increasing dominance in $A1$ are the same as those required to have action-reaction in $C2$ (namely [11] > 0 and [12] < 0). They imply that the low cost firm at the time when the second auction takes place will win that auction too, only if that firm is the firm with low costs at the beginning. But if firm 2 is the low cost firm at the time of the second auction, then it will lose that auction. Hence, no matter what happens in the first auction, firm 1 will win the second one.
However, it is the third condition that determines whether we have action-reaction or increasing dominance, because this condition gives us the winner of the first auction. Apart from the direct effect of the first auction, the effect induced by the existence of a second auction is:

$$\sigma_{II}(c,k_2) - \sigma_{II}(c,\Phi) + \pi^1_{II}(\Phi,k_2) + \pi^2_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) =$$
$$= \sigma_{II}(c,k_2) - \sigma_{II}(c,\Phi) + \sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) + \pi^1_{II}(\Phi,k_1) -$$
$$- \pi^1_{II}(\Phi,k_2)$$

This effect will be either negative or positive but with a small absolute value. Hence, only if the direct effect of the first auction is negative, i.e. favourable to firm 2, and relatively large in absolute value, we can expect action-reaction. Otherwise, increasing dominance will normally result when [11] > 0 and [12] < 0.

Cases $C1$ and $B2$ require [11] < 0 and [12] < 0. If these conditions are met, action-reaction occurs but not necessarily in the first auction. Note that in this case, the signs of [11] and [12] fulfil assumption 1 and if this assumption remains valid for the direct effect in the first auction, action-reaction is the only possible outcome. In fact, the induced effect here is,

$$\pi^2_{\text{II}}(k_1,\Phi) - \pi^1_{\text{II}}(\Phi,k_2) < 0$$

and hence, it is always favourable to firm 2, i.e. the initially high cost firm. If the direct effect in the first auction is favourable to the initially low cost firm and it can offset the induced effect, action reaction will occur but only after one period of increasing dominance. Obviously, assumption 1 ensures that the high cost firm will win the second auction.

The inverse to assumption 1 arises when [11] > 0 and [12] > 0 too, we can have increasing dominance, $A2$ or increasing dominance for firm 2 after action-reaction in the first period, $D2$. As Vickers points out, contrary to what might be expected, this is not a sufficient condition to have increasing dominance. Here, the direct effect in the second auction is always favourable to the firm that won the first one, and hence, it is just the opposite to what happens when [11] and [12] are both negative. However, the induced effect does not always reinforce the direct effect now. It is equal to:

$$[\sigma_{\text{II}}(\Phi,k_2) - \sigma_{\text{II}}(k_1,\Phi)] + [\pi^2_{\text{II}}(\Phi,k_2) - \pi^1_{\text{II}}(k_1,\Phi)]$$

which has no definite sign. Notice that the second term in brackets is always negative and this will counteract the positive direct effect. Ultimately, the sign of the induced effect will depend on the relative values of $\sigma_{\text{II}}(\Phi,k_2)$ and $\sigma_{\text{II}}(k_1,\Phi)$. If $\sigma_{\text{II}}(\Phi,k_2) < \sigma_{\text{II}}(k_1,\Phi)$, the induced effect will be negative and favourable to the second firm. Consequently, we can conclude that although the reverse to assumption 1 plays has a contrary influence over the direct effect than assumption 1, it is the induced effect what does not provide sufficiency to ensure increasing dominance[7].

Finally, if [11] < 0 and [12] > 0, cases $B1$ and $D2$ can arise. Here, the direct effect in the second auction is always favourable to firm 2, the initially high cost firm. The induced effect in the first auction is,

$$2\sigma_{\text{II}}(\Phi,c) - \sigma_{\text{II}}(k_1,\Phi) - \pi^1_{\text{II}}(k_1,\Phi) - \pi^2_{\text{II}}(k_2,\Phi) <$$
$$< 2\sigma_{\text{II}}(\Phi,c) - \sigma_{\text{II}}(k_2,\Phi) - \sigma_{\text{II}}(k_1,\Phi)$$

[7] As we will see in the second part of the paper, the way the induced acts on present auctions depends on the way the game is designed, and the reverse of assumption 1 can be a sufficient condition to guarantee increasing dominance in a modified game.

that can be positive or negative depending on the absolute value of the differences in [11] and [12].

From the discussion above some conclusions can be drawn already about the structure of this kind of models:

1. First, assumption 1 and its reverse are central to determine the sign of the direct effect, but, besides this effect,
2. the complete evolution of the market structure is complemented by the induced effect, and the influence of this effect depends on the way the game has been modelled.

In the next section we examine the implications of assumption 1 and in the rest of the paper we will consider alternative versions of our basic game to see how the influence of the induced effect varies accordingly.

5. Joint profits and firms' costs

As we have seen, the direct effect plays a crucial role in determining the outcome of the technology game, and this direct effect depends on how joint industry profits change when firm costs change. Under Cournot assumptions, and for a linear demand function, joint industry profits are an inverse function of the cost level of the high cost firm. Furthermore, zero profits in the high cost firm is a necessary condition for increasing dominance. Based on these two results. Vickers (1986) points out a certain relationship between rivalry in the product market and rivalry in the technology game. Using a particular version of assumption 1 we will explore the generality of Vickers' results using comparative statics.

From Dixit (1984) we can conclude that in any duopolistic structure with constant marginal costs $c_1 < c_2$ and product homogeneity, when the high cost firm changes its cost levels, assumption 1 can be expressed as

$$(d\sigma/dc_1) = (d\pi_1/dc_1) + (d\pi_2/dc_1) < 0 \qquad \text{with} \qquad [19]$$

$$(d\pi_1/dc_1) = -\Gamma^{-1}[p_2\, x_1\, a_1(v_1 - r_2)] - x_1 \qquad \text{and} \qquad [20]$$

$$(d\pi_2/dc_1) = \Gamma^{-1}[x_2\, p_1\, a_2(1 - r_2\, v_2)] \qquad [21]$$

where

$$\Gamma = \begin{vmatrix} \delta^2\pi_1/\delta x_1^2 & \delta^2\pi_1/\delta x_2^2 \\ \delta^2\pi_2/\delta x_1^2 & \delta^2\pi_2/\delta x_2^2 \end{vmatrix} > 0$$

$a_1 = \delta^2\pi_1/\delta x_1^2 < 0$; $a_2 = \delta^2\pi_2/\delta x_2^2 < 0$;[8]

v_i = conjectural variation for the ith firm,

x_i = output of the i^{th} firm,

$p_i = (\delta p(x_1, x_2)/\delta x_i)$

and r_i = is the slope of firm i's reaction function.

[8] These sign conditions are required by stability assumptions (see Dixit (1984)).

The cross effect ($d\pi_2/dc_1$) is always non-negative, but the effect of a change in the costs of the high cost firm on its own profits can go either way. There is a negative direct effect represented by $-x_1$, but there is an indirect effect acting via quantities that can reinforce or offset this direct effect.

It is easy to check that for a general Cournot-Nash equilibrium with constant marginal costs, $v_i = 0$ and

$$d\sigma/dc_1 < 0 \text{ if and only if } -p_2 x_1 a_1 r_2 + x_2 p_1 a_2 < x_1 \Gamma \qquad [22]$$

or

$$d\sigma/dc_1 > 0 \text{ if and only if } -p_2 x_1 a_1 r_2 + x_2 p_1 a_2 > x_1 \Gamma$$

With a linear demand function, the expressions above can be reduced to:

$$d\sigma/dc_1 \lesseqgtr 0 \text{ iff } 2(p_1)^2[x_1 - x_2] \lesseqgtr x_1 \Gamma,$$

but the left hand side of this expression is always negative because $x_1 < x_2$ and consequently, $d\sigma/dc_1$ is always negative as Vickers finds out. However, it is not possible to generalize this result from [22] above to any kind of demand function without any further elaboration. Note that in the left hand side of [22], the first term is negative while the second is positive while the right hand side of [22] is also positive.

As regards to the inverse of assumption 1, [19] suggests that it will usually arise in case of intense competition between the two firms in the product market, but this result is also subject to some qualifications. Under competition in that market, the conjectural variations will approach one as the product becomes perfectly homogeneous and then, $[v_1 - r_2]$ will be negative as Dixit proves for the normal case when $r_2 < 0$. As a result, [20] will have a positive component and a negative one. [21] will not have any influence here as it will be equal to zero.

The former discussion points out that there is some kind of link between market structure and the sign of the direct effect in the technology game. However, this link cannot be generalized is the sense that Vickers does when he establishes that

> «(the conjectural variation) is regarded by some people as an index of 'competitiveness' since collusion is approached as the conjectural variation approaches 1 and Bertrand behaviour is approached as the conjectural variation approaches-1. In this language, it can be stated that in a single patent race, the incentive of the high cost firm minus the incentive of the low cost firm increases the behaviour in the product market becomes less competitive.» (...) «... assumption 1 is likelier to hold as behaviour in the market is less competitive». [Vickers (1986) p. 11].

Demand conditions seem to play an important role in the sign of the direct effect as can be seen from expression [19], although the conjectural variations also have an important influence. However, simulations with a linear demand function $p = 100 - x_1 - x_2$, $c = 20$ and $v_1 = v_2 = 0{,}5$ show that joint industry profits fall when the costs of the high cost firm fall from 39 to 37 but increase if they fall from 23 to 21 for instance.

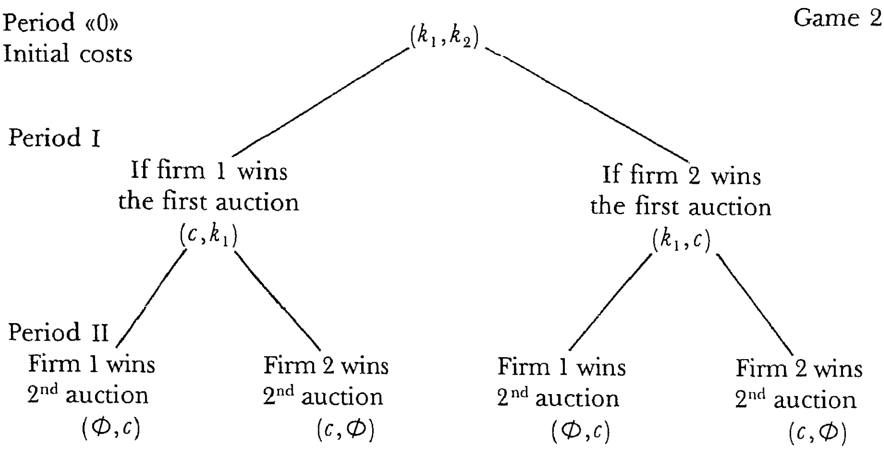
This kind of results, besides product differentiation[9], could help to explain the different results obtained by Vickers and Beath *et al.* But in sequential models of innovation, the induced effect plays a crucial role in the outcome of the technology game. In the next sections, we present a series of variations to the basic model proposed by Vickers in order to study how his conclusions are modified as the structure of the game varies.

6. Sequential innovations with free diffusion

It has been argued often that one of the market imperfections associated with the technology markets is due to the low cost of transmission of the information, which facilitates almost free diffusion of some technologies. This has been considered as factor playing in favour of the high cost firms and countries. However, if we introduce free diffusion in game 1, the structure of the game is simplified and the influence of future innovations is lost. In other words, with free diffusion there is no difference between a game with a sequence of innovations and a sequence of games with only one innovation each. Nevertheless, as an anonymous referee has pointed out to us, this model is also valid for any number of time periods, as long as the patents give an exclusive access to the new technology but just for one period of time.

In this setting, the action-reaction equilibrium is as likely as any other and it does not depend on the assumption 1.

We will assume again the same game structure as in game 1. However, we will assume that at the beginning of each period, the high cost firm can use for free the technology abandoned as obsolete by its rival. This gives us the following game 2,



[9] Note that under product differentiation, conjectural variations are not symmetric in the competitive case as they will be $v_i = -(p_i^i / p_j^i)$.

Note that now, only two final price structures remain but the number of possible equilibria is still four. If the final price structure is (Φ,c) the payoffs of the second period to firm 1 are,

$$\mu_1 = \sigma(\Phi,c) - \pi^2_{II}(c,\Phi)$$

and to firm 2 are,

$$\mu_2 = \pi^2_{II}(\Phi,c)$$

but if the final price structure is (c,Φ),

$$\mu_1 = \pi^1_{II}(c,\Phi) \quad \text{and} \quad \mu_2 = \sigma(c,\Phi) - \pi^1_{II}(\Phi,c)$$

From here, it is easy to define the reservation prices for the first innovation as,

$$P^1_I = \pi^1_I(c,k_1) + \max[\sigma(\Phi,c) - \pi^2_{II}(c,\Phi),\ \pi^1_{II}(c,\Phi)] - \pi^1_I(k_1,c) \\ - \max[\sigma(\Phi,c) - \pi^2_{II}(c,\Phi),\ \pi^1_{II}(c,\Phi)]$$

$$P^2_I = \pi^2_I(k_1,c) + \max[\sigma(\Phi,c) - \pi^1_{II}(\Phi,c),\ \pi^1_{II}(\Phi,c)] - \pi^2_I(c,k_1) \\ - \max[\sigma(\Phi,c) - \pi^1_{II}(\Phi,c),\ \pi^2_{II}(\Phi,c)]$$

Conditions to know which firm will win the first innovation are derived from the difference between P^1_I and P^2_I, but

$$P^1_I - P^2_I = \sigma(c,k_1) - \sigma(k_1,c) = 0$$

Thus, no firm shows a reservation price different from its rival's in any auction and who will win is undetermined. Note that the initial asymmetry between firms is lost after one period and has no influence on the outcome of the game.

In terms of our discussion in section IV, what free diffusion introduces is a substantial difference in the technology game. First, the induced effect is the same for both firms. But second, this is due to the fact that, with free diffusion the technology game is no longer a patent race: the firm losing one auction can still improve its technology, not as much as the winner, but it will have lower production at no cost at all. Using a terminology introduced by Dasgupta, firms are indifferent between fighting for the patent in the auction and waiting.

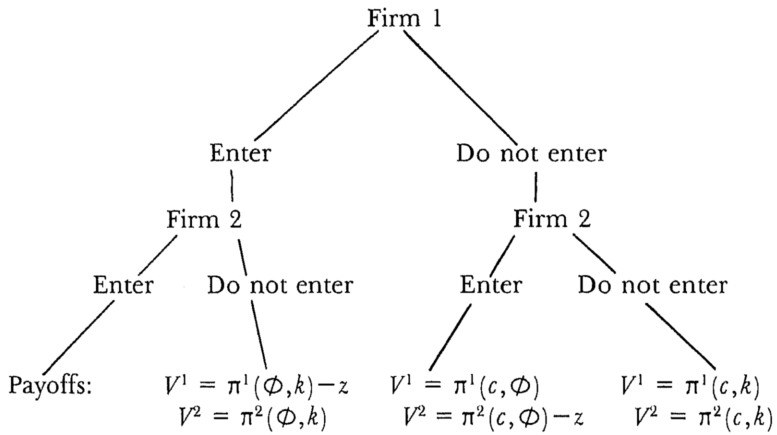
Finally, note that action-reaction still remains as a possible equilibrium, but assumption 1 has no influence on its appearance. However, free diffusion ensures that increasing dominance will exist only if $k_2 - k_1 < c - k_1 < \Phi - c$.

7. Sequential innovation with non-reversible R&D costs

As we pointed out before, in the models by Vickers and Beath *et al.* R&D costs are reversible as the firm not winning the patent does not incur into any R&D

cost. This is not the case in real life as any firm competing for an innovation has to incur into positive R&D costs while only the winner will obtain some reward for these expenses. Furthermore, in the models mentioned above, firms always enter the patent race. This is logical when this is free, but if there is some non-reversible cost in joining the patent race, firms will have to decide first whether to enter or not the patent race.

The structure of the game is very simple. Each firm has to pay a fixed amount z to enter the patent race. If both firms joint the race, the firm with the highest reservation price wins it, and that will depend on $\sigma(\Phi,k) \lessgtr \sigma(c,\Phi)$. But in that case, the firm losing the race has to pay the R&D cost z. If one firm joints the race and the other does not, then the first one will win it at a cost equal to z, but the other firm would save that amount. If none of the two firms enter the race, the innovation is not adopted. It is easy to see that a solution with both firms entering the patent race is not a Nash equilibrium because whatever is the sign of $\sigma(\Phi,k) - \sigma(c,\Phi)$, the firm losing the auction would win z by not joining the race. The only possible Nash equilibria are:



If $\sigma(\Phi,k) > \sigma(c,\Phi)$
$V^1 = \pi^1(\Phi,k) - \pi^2(c,\Phi) + \pi^2(\Phi,k)$
$V^2 = \pi^2(\Phi,k) - z$

If $\sigma(\Phi,k) < \sigma(c,\Phi)$
$V^1 = \pi^1(c,\Phi) - z$
$V^2 = \pi^2(c,\Phi) - \pi^1(\Phi,k) + \pi^1(c,\Phi)$ with $\Phi < c < k$

— firm 1 enters and 2 doesn't if $\sigma(\Phi,k) > \sigma(c,\Phi)$ and if $\pi^1(\Phi,k) - z > \pi^1(c,k)$

— firm 2 enters and firm 2 does not if $\sigma(\Phi,k) < \sigma(c,\Phi)$ and if $\pi^2(c,\Phi) - z > \pi^2(c,k)$

The second condition above is important to avoid no firm joining the race. If the single period game above is expanded to the general game 1 with three periods, we get the following reservation prices for the two firms in period 1:

$$P_{I}^{1}=\pi_{I}^{1}(c,k_2)+\max\left[\pi_{II}^{1}(\Phi,k_2)-z,\pi_{II}^{1}(c,\Phi)\right]-\pi_{I}^{1}(k_1,c)-\max[\pi_{II}^{1}(\Phi,c)-z,\ \pi_{II}^{1}(k_1,\Phi)]$$

$$P_{I}^{2}=\pi_{I}^{2}(k_1,c)+\max[\pi_{II}^{2}(k_1,\Phi)-z,\ \pi_{II}^{2}(\Phi,c)]-\pi_{I}^{2}(c,k_2)-\max[\pi_{II}^{1}(c,\Phi)-z,\ \pi_{II}^{1}(\Phi,k_2)$$

Note that the second condition implies that the first terms in the brackets are maxima because if we require that $\pi^1(\Phi,k)-z>\pi^1(c,k)$ and $\pi^2(c,\Phi)-z>\pi^2(c,k)$ the former implies that,

$$\pi_{II}^{1}(\Phi,k_2)-z>\pi_{II}^{1}(c,\Phi)\ \text{because}\ \pi_{II}^{1}(c,k)>\pi_{II}^{1}(c,\Phi)$$

and

$\pi_{II}^{2}(k_1,\Phi)-z=\pi_{II}^{1}(\Phi,k_1)-z>\pi_{II}^{2}(\Phi,c)=\pi_{II}^{1}(c,\Phi)$ and $\pi_{II}^{1}(c,k)>\pi_{II}^{1}(c,\Phi)$ for any $k<c$[10].

Subtracting P_I^2 from P_I^1 we get,

$P_I^1-P_I^2=\sigma(c,k_2)-\sigma(c,k_1)+[\pi_{II}^{1}(\Phi,k_2)-\pi_{II}^{1}(\Phi,k_1)]$ where the term in brackets at the right hand side is always positive. Consequently we can conclude that the reverse to assumption 1. i.e. $\sigma_t(c,k+\varepsilon)>\sigma_t(c,k)$ is a sufficient condition for *increasing dominance.*

This result, contrary to Vickers'conclusions, is due to the new structure of the game. Notice that while the induced effect was acting in favour of the second firm in Vickers'model, here, it is always favourable to firm 1. As a result, action-reaction is possible if and only if the direct effect act in the opposite direction and is big enough to compensate the induced effect.

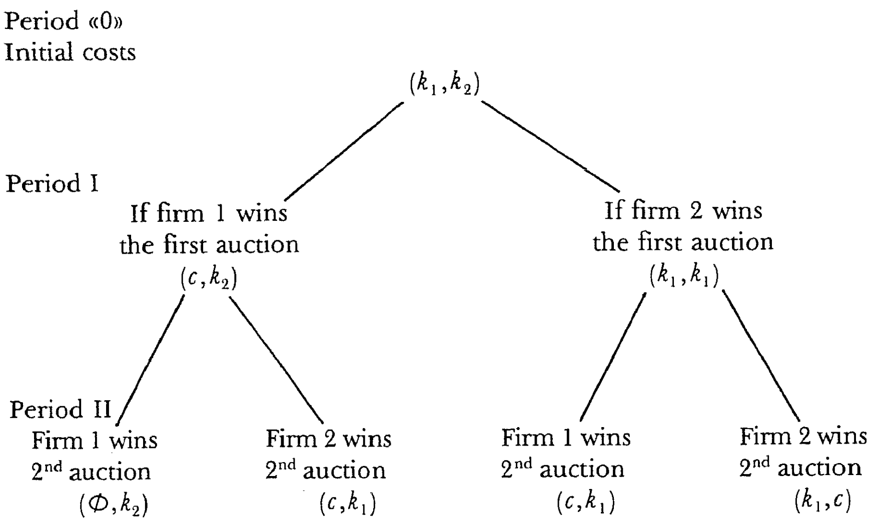
8. Differences in the ability to adopt new technologies («history matters»)

As we have mentioned before, in game 1 both firms have the same opportunities to have access to any innovation that is auctioned at the beginning of each period. This is one of the most striking characteristics of Vickers' model. In a model with $n+1$ periods and n innovations, it is hard to envision that a firm that has lost $n-1$ auctions and is technologically far behind its rival, can win the last auction and become the low cost firm overnight.

In game 3 below, we have incorporated some kind of influence of the relative backwardness of a firm. This can be done in several ways. One can

[10] Similarly, $\pi^2(c,\Phi)-z>\pi^2(c,k)$ implies that $\pi_{II}^{1}(\Phi,c)-z>\pi_{II}^{1}(k_1,\Phi)$ and $\pi_{II}^{1}(c,\Phi)-z>\pi_{II}^{1}(\Phi,k_2)$; $\pi^2(c,k)>\pi^2(\Phi,k_1)>\pi^2(\Phi,k_2)$ for any k.

Game 3



think of some extra costs that the high cost firm has to pay for the patent in which the low cost firm does not have to incur. Alternatively, one can think of a situation in which the high cost firm can not have access to the «latest» technology at the beginning of each period, but to the immediately more advanced technology to the one it is using. Note that now, if firm 2 wins the first auction, symmetry is restored and firms have the same incentive to win the second patent.

A particular case of this game is the case in which the innovation reduces costs in each firm by a certain fixed amount that is subtracted from its past constant marginal costs, i.e. $k_2 - k_1 = k_1 - c = c - \Phi$.

Following the usual procedure we find that the difference in reservation prices in the first period will be

$$P_I^1 - P_I^2 = \sigma(c,k_2) + \max[\sigma(\Phi,k_2) - \pi_{II}^2(c,k_1), \pi_{II}^1(c,k_1)] - \sigma(k_1,k_1) - \max[(c,k_1) - \pi_{II}^2(k_1,c), \pi_{II}^1(k_1,c)] - \max[\sigma(k_1,c) - \pi_{II}^1(c,k_1), \pi_{II}^2(c,k_1)] + \max[\sigma(c,k_1) - \pi_{II}^1(\Phi,k_2), \pi_{II}^2(\Phi,k_2)]$$

The first interesting result that arises here is that assumption 1 and its reverse have no influence on the direct effect while asymmetry is preserved. Note that the direct effect in the first period depends on the sign of $\sigma(c,k_2) - \sigma(k_1,k_1,)$, i.e. how joint industry profits vary when they start to diverge from a certain common level. It can be proved that with a linear demand function that difference is positive if $k_1 - c = k_2 - k_1$ both under Cournot and under Bertrand conjectures. However, this is not necessarily true when $k_1 - c \neq k_2 - k_1$. Hence, a Cournot behaviour in the product market play in favour of the first firm in the first auction, not leading to action-reaction necessarily.

The induced effect has no definite sign. If $\sigma(\Phi,k_2) - \sigma(c,k_1) > 0$, it is just

$$\pi^1(\Phi,k_2) - \pi^2(k_1,c) + 2\pi^2(\Phi,k_2) - 2\pi^2(c,k_1)$$

However, if $k_2 - k_1 = k_1 - c = c - \Phi$ it will usually be positive under Cournot behaviour producing increasing dominance as a result.

9. Conclusion

In this paper we have studied the structure and limitations of a series of models of sequential innovation that try to explain market evolution. For that purpose, a very simplified version of the model is presented. Two kinds of incentives to win each auction are distinguished: the direct effect that exists in any single period game and the induced effect. In the original model used by Vickers and others, the first one depends on how joint industry profits vary when the costs of the high-cost firm change. However, this is not true when «history matters». Then, the direct effect depends on how industry profits vary when costs diverge.

The induced effect captures the influence of future auctions on the willingness to pay for the patent currently being auctioned. If depends heavily on the structure of the game, particularly when there is free diffusion and when R&D costs are non-reversible.

Appendix

We can assume either:

i) $\sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) > 0$ and $\sigma_{II}(k_1,\Phi) - \sigma_{II}(\Phi,c) < 0$,

then [13]

$$P_I^1 - P_I^2 = \sigma_I(c,k_2) - \sigma_I(k_1,c) + \sigma_{II}(\Phi,k_2) - 2\sigma_{II}(\Phi,c) + \pi_{II}^2(k_1,\Phi) + \pi_{II}^2(\Phi,k_2)$$

Under assumption (*i*), firm 1 will always win the second auction and depending on the sign of [13] we could have type *A* or type *C* equilibria. Both are possible because (*i*) implies,

$$\sigma_I(c,k_2) - \sigma_I(k_1,c) > 0$$
$$\sigma_{II}(\Phi,k_2) - \sigma_{II}(\Phi,c) > 0$$

but $\pi_{II}^2(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) - \sigma_{II}(\Phi,c) < 0$ because,

$$\pi_{II}^2(k_1,\Phi) + \pi_{II}^2(\Phi,k_2) < \pi_{II}^1(\Phi,k_1) + \pi_{II}^2(\Phi,k_1) = \sigma_{II}(\Phi,k_1) < \sigma_{II}(\Phi,c)$$

and the sign of [13] could go either way.

ii) $\sigma_{II}(\Phi,k_2) - \sigma_{II}(c,\Phi) > 0$ and $\sigma_{II}(k_1,\Phi) - \sigma_{II}(\Phi,c) > 0$, then [14]

$$P_I^1 - P_I^2 = \sigma_I(c,k_2) - \sigma_I(k_1,c) + \pi_{II}^2(\Phi,k_2) - \pi_{II}^1(k_1,\Phi) - \sigma_{II}(\Phi,k_2) - \sigma_{II}(k_1,\Phi)$$

Here, (*ii*) implies that the firm that won the first auction will win the second one too. Hence, cases *A* and *D* are the only two possible outcomes depending on the sign of [14], which is uncertain because although $[\pi^2_{\mathrm{II}}(\Phi,k_2) - \pi^1_{\mathrm{II}}(k_1,\Phi)]$ is always positive, (*ii*) does not imply any sign on $\sigma_{\mathrm{II}}(\Phi,k_2) - \sigma_{\mathrm{II}}(k_1,\Phi)$.

iii) $\sigma_{\mathrm{II}}(\Phi,k_2) - \sigma_{\mathrm{II}}(c,\Phi) < 0$ and $\sigma_{\mathrm{II}}(k_1,\Phi) - \sigma_{\mathrm{II}}(\Phi,c) < 0$, then [15]

$$P^1_{\mathrm{I}} - P^2_{\mathrm{I}} = \sigma_{\mathrm{I}}(c,k_2) - \sigma_{\mathrm{I}}(k_1,c) + \pi^2_{\mathrm{II}}(k_1,\Phi) - \pi^1_{\mathrm{II}}(\Phi,k_2)$$

Here, (*iii*) implies that the firm that lost the first patent will always wins the second auction. Consequently, only cases *B* and *C* are possible. Furthermore,

$$\pi^2_{\mathrm{II}}(k_1,\Phi) - \pi^1_{\mathrm{II}}(\Phi,k_2) = \pi^1_{\mathrm{II}}(\Phi,k_1) - \pi^1_{\mathrm{II}}(\Phi,k_2) < 0.$$

As a result, if assumption 1 holds for any period, only case *C* can occur. But if the reverse of assumption 1 holds for the first auction, case *B* may appear.

iv) $\sigma_{\mathrm{II}}(\Phi,k_2) - \sigma_{\mathrm{II}}(c,\Phi) < 0$ and $\sigma_{\mathrm{II}}(k_1,\Phi) - \sigma_{\mathrm{II}}(\Phi,c) > 0$, then [16]

$$P^1_{\mathrm{I}} - P^2_{\mathrm{I}} = \sigma_{\mathrm{I}}(c,k_2) - \sigma_{\mathrm{I}}(k_1,c) + 2\sigma_{\mathrm{II}}(\Phi,c) - \sigma_{\mathrm{II}}(k_1,\Phi) - \\ - \pi^1_{\mathrm{II}}(k_1,\Phi) - \pi^1_{\mathrm{II}}(\Phi,k_2).$$

In this case, firm 2 always wins the second auction. However, nothing can be said with certainty about the sign of [16].

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Abstract

This paper examines the structure of several recent models that study the relationship between the sequential introduction of innovations and the market structure of the industry where they take place. To make the analysis simple and tractable we present some —different but related— duopoly models with three periods and two possible innovations. We pay particular attention to the study of J. Vicker's recent proposition about the relationship between the patent game and the market game for the final product. Namely, that the levels of rivalry in the market game for the final good, will determine the equilibria of the patent game.

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