## THE ECONOMETRICS OF THE STOCK MARKET I: RATIONALITY TESTS

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This paper discusses some of the most significant contributions to the empirical literature on stock market rationality. The main concern of this literature is whether discounted excess returns are forecastable under a maintained assumption about the discount factor. It has concentrated on the time series properties of the aggregate stock market.

There is now overwhelming evidence that the simple present value model for stock prices with constant expected returns is rejected by the data. The disagreement is now centred on whether the observed predictability is due to rational variation in expected returns or the result of market inefficiencies.

## 1. Introduction

The empirical study of the stock market constitutes one of the research topics that has received most vivid interest in the last fifteen years. Being at the interface between macroeconomics and finance, these studies have combined the approaches employed in the two disciplines.

The applied finance literature of the early seventies broadly supported the view that the stock market as a whole was efficient, and that the static Capital Asset Pricing Model constituted a valid representation of the relative valuation of many assets. In contrast, the late seventies and early eighties witnessed the development of alternative asset pricing models, and the appearance of disturbing evidence questioning the rationality of the market.

By and large, both strands of the literature grew separately, and only at the end of the eighties a consensus view that these questions constitute two sides of the same coin emerged. Such a duality can be best seen in terms of the basic equilibrium relation for stock returns:

$$E_{\iota-1}(R_{j\iota}\zeta_{\iota})=1$$

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where  $R_{ji}$  is the one period (gross) holding return over period t on a stock bought at the end of period t-1 and  $\zeta_i$  a common stochastic discount factor which discounts uncertain payoffs differently across different states of the world (see part II of this review). The rationality literature main concern is whether  $R_{ji}\zeta_i-1$  is forecastable on the basis of variables in the information set under a maintained assumption about  $\zeta_i$ , usually  $\zeta_i=\zeta \ \forall t$ . It has concentrated on the time series properties of the aggregate stock market. On the other hand, the empirical asset pricing literature focuses on the validity of a specific model for  $\zeta_i$  by testing if  $E(R_{ji}\zeta_i-1)=0$  for several assets simultaneously. Traditionally, its main emphasis has been on cross-sectional aspects, although recently it has been shifting towards intertemporal models, in which agents actions are based on the conditional distribution of returns. This is largely motivated by the fact that the changing volatility of financial markets is nowadays well documented and widely recognised as one of their main characteristics.

The purpose of this survey is to discuss just a few of what I regard as the most significant contributions to this dual literature. In the first part, I discuss rationality tests. In the second, asset pricing theories.

Several other very active areas of research in the applied financial econometrics literature (such as continuous time models, portfolio allocation and evaluation, volatility measures or market microstructure) had to be excluded. (Un)fortunately, there is scope for parts III, IV, ...

## 2. Definition

The definition of stock market rationality is usually implicit rather than explicit. It involves two important conditions:

- a) There is informational efficiency.
- b) Prices reflect fundamental value.

The first condition says that asset prices must incorporate the available information in such a way that no abnormal profit opportunities arise by using that information alone. Notice that the above definition is information dependent, in the sense that different information sets will lead to different concepts of informational efficiency. In practice, three main forms are usually considered (see Fama (1970, 1991)):

- Weak efficiency, when the information used contains only lagged values of the price, dividend or return series, and/or «macroeconomic» information such as past interest rates, or month of the year.
- Semi-strong efficiency, when publicly available information specific to the firm under consideration is also used.
- <sup>1</sup> This idea is often mistaken with rationality of the market, but it is only a necessary condition. As we shall see below, there are instances in which a) holds but b) does not.

Strong efficiency when all potential information, including private one, is considered.

Strong efficiency is too a restrictive condition, for if prices reflect all potential information, the incentives to obtain this information may disappear. This theoretical issue has been considered by Grossman and Stiglitz (1980) amongst others. In this review, we shall concentrate mainly on weak efficiency tests. Semi-strong efficiency is usually tested by means of the so-called event studies, and an extensive set of references can be found in Fama (1991).

Informational efficiency was discussed in financial economics long before the Rational Expectations revolution took place in micro and especially macroeconomic theory. However, it amounts to the same thing: the subjective expectations of the agents coincide with the mathematical expectation of the true stochastic model. In the stock market, it implies that for risk neutral arbitrageurs, no opportunities of extraordinary profits arise by using publicly available information.

The second requirement has often been ignored in the discussion. But most casinos are also informationally efficient, and yet, they attract little attention from agents when taking their investment decisions. The link to fundamental (i.e. «correct») value is paramount to justify the stock market as a means of redistributing risks in the economy and providing the right indicators of value.

In order to make the distinction between a) and b) clearer let us introduce some notation. Let  $R_t$  be the one period (gross) holding return over period t on a stock bought at the end of period t-1. Let  $D_t$  be the dividend paid at the end of period t and let  $P_t$  be the ex-dividend price at the end of the same period. By definition.

$$R_t \equiv (D_t + P_t)/P_{t-1}$$

In order to derive an expression for  $P_{\iota_1}$ , let us make the assumption of rational expectations. Informational efficiency implies that the return required by the stock market participants,  $V_i$ , must be equal to the conditional expectation of  $R_i$ ,  $V_i = E(R_i|I_{\iota_1}) = E_{\iota_1}(R_i)$ , where  $I_{\iota_1}$  is (the sigma algebra generated by) all publicly available information at the end of period t-1 (which includes the equilibrium value of  $P_{\iota_1}$ ). That is,

$$v_{i}^{\epsilon} = v_{i} = E_{\iota 1}[(D_{i} + P_{i})/P_{\iota 1}]$$

This is an equilibrium condition. It implies that given  $I_{t-1}$ , the average return obtained by holding the asset must be exactly what agents required. As a consequence,

$$P_{\iota 1} = E_{\iota 1} [(\nu_{\iota}^{-1} D_{\iota}) + (\nu_{\iota}^{-1} P_{\iota})]$$

Since  $P_t = E_t[(v_{t+1}^{-1} D_{t+1}) + (v_{t+1}^{-1} P_{t+1})]$ , by the law of iterated expectations

$$P_{\iota_{1}} = E_{\iota_{1}}(\nu_{\iota}^{-1} D_{\iota}) + E_{\iota_{1}}[(\nu_{\iota}\nu_{\iota+1})^{-1} D_{\iota+1}] + E_{\iota_{1}}[(\nu_{\iota}\nu_{\iota+1})^{-1} P_{\iota+1}]$$

By repeated substitution we will have that

$$P_{\iota + 1} = E_{\iota + 1} \left[ \sum_{k=0}^{K} \left( \prod_{j=0}^{k} \nu_{\iota + j} \right)^{-1} D_{\iota + k} \right] + E_{\iota + 1} \left[ \left( \prod_{j=0}^{K} \nu_{\iota + j} \right)^{-1} P_{\iota + K} \right]$$

If we assume that the first series is convergent (which requires the rate of growth in dividends be smaller than the required rates of return) and impose the transversality condition

$$\lim_{K\to\infty} E_{i-1} \left[ \left( \prod_{j=0}^K \nu_{i+j} \right)^{-1} P_{i+K} \right] = 0,$$

we end up with the well-known Present Value formula for stock prices:

$$P_{i-1} = E_{i-1} \left[ \sum_{k=0}^{\infty} \left( \prod_{j=0}^{k} v_{i+j} \right)^{-1} D_{i+k} \right]$$

It says that stock prices are the present discounted value of rationally forecasted dividends. The right hand side is usually referred to as the fundamental value of the stock.

How can then be the share price different from its fundamental value and at the same time be informationally efficient? The answer is that the equilibrium condition has infinite solutions, the fundamental value being the only one that satisfies the transversality condition (see Blanchard and Watson (1982)). In particular, let  $P_{l-1} = P_{l-1} + B_{l-1}$ . Then, provided that  $B_{l-1} = E_{l-1}(\nu_l^{-1}B_l)$ ,  $P_{l-1}^{*}$  will also safisfy the equilibrium condition, but certainly not the tranversality condition. The discrepancy term  $B_{l-1}$ , is called a «rational» bubble. It is rational because informational efficiency still prevails, but it certainly introduces a deviation of the share price from its fundamental value. Hence, if stock prices contained a rational bubble, the market would not be rational, only informationally efficient.

Bubbles are an example of self-fulfilling expectations: they affect share prices because everybody expects them to do so. The most simple example of a bubble is a deterministic one, i.e.  $B_t = v_t B_{t-1}$ . However this is quite unlikely to be consistent with actual stock prices behaviour, since it would imply a continuously growing process for the price. Stochastic bubbles are in principle more plausible. West (1988b) provides a simple example. In each period the bubble either floats (with probability  $\pi_t$ ), or bursts. If the bubble floats, it grows at the rate  $v_t/\pi_t > v_t$ . Investors receive an extraordinary return to compensate them for the capital loss that would have occurred had the bubble burst. However, the bubble is not necessarily an extraneous event. For example, it may be that the bubble bursts if there is bad news about dividends (e.g. the intrinsic bubbles in Froot and Obstfeld (1991)). On the other hand it could be related to «sunspots».

Nevertheless, there are some theoretical restrictions on the behaviour of bubbles. Since  $E_{\iota_1}(B_{\iota_1\iota}) = B_{\iota_1} \cdot E_{\iota_1} \left( \prod_{j=0}^{k} \nu_{\iota_1j} \right)$ , it is clear that on a freely disposable asset a bubble cannot be negative or otherwise the price will become negative eventually (see Diba and Grossman (1987)). Also, if the bubble is ever 0, it will always be 0, so if there is a bubble it must have

been there forever and it will be there forever. Bubbles are thus an empty box, since there is no explanation of how they started. Finally, if agents are infinitely lived, a bubble cannot be positive either, for it is not consistent with general equilibrium considerations (see Tirole (1982)). However, in an overlapping generations model it might be possible to have bubbles provided every generation is willing to buy shares at a price in excess of fundamental value and the rate of return on the asset is smaller than the rate of growth of the economy (see Tirole (1985)).

Most earlier tests of stock market rationality assumed that  $v_t = v$  for all t. The main reason was tractability of the present value formula. Under this assumption the present value formula becomes simply

$$P_{t-1} = \sum_{k=0}^{\infty} \nu^{-(k+1)} E_{t-1} (D_{t+k})$$

The constant expected returns formulation is sometimes known as the «martingale model» (see LeRoy (1989)). Such a name derives from the fact that the value of a mutual fund that held the stock and reinvested all dividend payments in further share purchases would be a martingale when discounted to time 0. But even in the «martingale model» it is not true that stock market efficiency implies that  $P_t$  follows a random walk. This common misconception is based more on the earlier empirical evidence than in any well based theory of stock market behaviour. As a (useful) counter-example let us make the rather strong assumption that  $E_{t,1}(D_t) = \gamma D_{t,1}^2$ . By the properties of the AR(1) process,  $E_{k1}(D_{t+k}) = \gamma^{k+1} D_{k1}$ . Hence, it is not difficult to see that  $P_{\nu 1} = \gamma (\nu - \gamma)^{-1} D_{\nu 1} = (\nu - \gamma)^{-1} E_{\nu 1} (D_t)$ , so that  $P_{\nu 1} = \gamma^{-1} E_{\nu 1} (P_t)$ , i. e.  $P_t$  is also an AR(1) process with the same parameter. As a consequence  $E_{k1}(\Delta P_l) = (\gamma - 1)P_{k1}$  so that unless  $\gamma = 1$ , price changes are predictable. The intuition is that efficiency requires  $E_{t,l}(R_t - \nu) = 0$ , not  $E_{t,l}(\Delta P_t) = 0$ . In fact, given the assumed dividend process, the predictability of share prices ensures that no abnormal returns can be made.

# 3. Empirical Tests of the Present Value Formulation with Constant Expected Returns

We shall discuss next the empirical evidence accumulated over the last fifteen years against the simple present value model for stock prices with constant discount rates. In order to follow the heated academic debate over its validity, the presentation of the tests follows a chronological order more closely than might otherwise be desirable.

## 3.1. Excess Volatility Tests

By and large, the applied finance literature of the early seventies supported the view that the stock market as a whole was efficient, and furthermore,

<sup>2</sup> This does not simply say that the univariate Wold representation of  $D_t$  is an AR(1) process, bur rather, that given  $D_{t1}$ , any other information is redundant in forecasting dividends.

that constant expected returns constituted a very plausible assumption. Although there always was some anomalous empirical evidence put forward by an atheistic minority, the line of attack to the rationality of the stock market which caused a major impact amongst economists was the initial volatility tests of Shiller (1981) and LeRoy and Porter (1981). The rationale for these tests is that forecasts should be less volatile (i.e. have a smaller variance) than actual outcomes. To understand this consider randomly drawing a number from a normal (0,1) distribution every period. Conditional on past outcomes, the expected value is always 0. The variance of the forecast is 0 whereas the variance of the actual outcome is 1. In the stock market context, let us define the «perfect foresight» share price,  $P_{i-1}^*$  as the present value of actual future dividends, i. e.

$$P_{t-1}^* = \sum_{k=0}^{\infty} v^{-(k+1)} D_{t+k}$$

Obviously  $P_{\iota_1}^*$  is not known at period  $\iota$ -1, but stock market rationality implies that  $P_{\iota_1} = E_{\iota_1}(P_{\iota_1}^*)$ , i. e. the share price is the rational forecast of  $P_{\iota_1}^*$ . Defining the forecast error as  $u_{\iota_1} = P_{\iota_1}^* - P_{\iota_1}$ , the optimality of the forecasts implies that the orthogonality condition  $E_{\iota_1}(u_{\iota_1}) = 0$  must be satisfied, for otherwise there would be valuable unexploited information in  $I_{\iota_1}$ . In particular,  $E_{\iota_1}(u_{\iota_1}P_{\iota_1}) = 0$  so that rational forecast and forecast error are (un)conditionally uncorrelated. Since  $P_{\iota_1}^* = P_{\iota_1} + u_{\iota_1}$ , assuming the appropriate unconditional moments are bounded,  $V(P_{\iota_1}^*) = V(P_{\iota_1}) + V(u_{\iota_1})$ , and  $V(P_{\iota_1}^*) \geq V(P_{\iota_1})$ . This simple result is the basis of the variance bounds test. A nontrivial advantage of dealing with unconditional moments is that there is no need to characterise  $I_{\iota_1}$ . Hence, the test is not sensitive to differential information between agents and econometricians.

Nevertheless, the implementation of the tests poses several practical problems. First, the computation of  $P_{t-1}^*$  involves an infinite sum of future dividends. Since any sample if finite, some terminal condition, such as  $P_T^* = P_T$  must be used. Besides, stock market prices are generally upward trending over a long time interval, so the (time series) sample variances of the raw data tend to increase with the sample size. Shiller's solution was to assume that  $D_t$ , and hence  $P_t$ , randomly fluctuated around a deterministic exponential trend, and computed the variances for the detrended data. His results were devastating: over a century of US data, the sample standard deviation of detrended prices was more than 5 times as large as the standard deviation of detrended experfect foresights prices.

Such a result shocked many people. The criticisms did not take long to come. Shiller (1981) tested market rationality under the maintained assumptions that expected returns are constant and the dividend process is stationary around an exponential trend. As is true of virtually all econometric tests of a particular hypothesis, it is therefore a joint test, and as such, rejection could be due not only to a failure of market rationality, but also from a failure of the maintained assumptions.

From the econometric point of view the major initial criticism was the issue of whether  $D_i$  is indeed fluctuating around a deterministic trend or whether

it has a unit root. For if the latter is true, the population variances are infinite, and hence any relationship between (obviously) finite sample counterparts is meaningless. As an example, Marsh and Merton (1986) proposed a model in which managers smooth dividend payments to maintain a long run relation to permanent earnings. For instance, managers may set  $D_t = \lambda P_{t-1}$ . In this context, the variance inequality will be reversed in terms of sample moments since  $P_{t-1}^*$  is a weighted average of present and future prices. As Shiller (1986) pointed out, the main rationale of this result is that the assumed dividend process is now endogenous with a unit root.

Kleidon's (1986a) criticisms pointed in the same direction. He criticized eye-ball testing by recalling that the ensemble variance (i.e. the variance across different realizations of the underlying stochastic process) is not the same thing as the time-series variance for non-ergodic series. If dividends had a unit root, Shiller's deterministically detrended series would suffer exactly from such a problem, and the apparent implication from his famous plot that actual prices were too volatile would be inadequate.

The second main econometric criticism was related to finite sample problems. Flavin (1983) and Kleidon (1986b) raised the issue forcefully. The finite sample problems are due to both the serial correlation in  $P_{t-1}^*$  and to the terminal condition. To see why the serial correlation is a problem in the constant expected returns framework recall the AR(1) model for dividends. Then we saw that  $P_{t-1}$  is proportional to  $D_{t-1}$ , and hence  $P_{t-1}^*$  is a moving average of future prices. As a result, the sample estimate of its variance is more downward biased in finite samples than the sample estimate of the variance in  $P_{t-1}^*$ .

The terminal condition also creates problems. Suppose that a company does not pay any dividends at all during the sample period. Nevertheless  $P_{i-1}$  moves a lot as news about future profit opportunities unwind. Our  $P_{i-1}^*$  computed under the assumption that  $P_T^* = P_T$  will be a simple exponential trend, so that the detrended series will be a constant with 0 variance.

But by far the most important criticism of the excess volatility tests is the assumption of constant expected returns as there is always a series of  $\nu_t$  such that the behaviour of  $P_{t-1}^*$  can be made consistent with that of  $P_{t-1}^{-3}$ . An example of how changes in  $\nu_t$  may affect the volatility tests is the so-called peso problem. Suppose that the stock market is preoccupied with events of dramatic consequences but very low probability, e.g. nuclear war. Since this has not happened,  $P_{t-1}^*$  has not been affected, but if these perceived probabilities have changed then  $P_{t-1}$  would have. This is also implicitly related to the finite sample problem, for if the sample covers a very long time span, the probabilities of such an event must be very low indeed for it not to happen.

<sup>&</sup>lt;sup>3</sup> In part II of this survey, we shall see how it can be achieved for a finite number assets simultaneously

Grossman and Shiller (1981) attempted to model the variation in  $\nu_i$  more formally by using the Consumption Capital Asset Model (see section 4 of the second part of this survey). Under the assumption that the representative individual's lifetime utility function is of the constant degree of relative risk aversion (CRRA) type (i.e. isoelastic), the stochastic discount factor  $\zeta_i$  will be given by  $\rho^{-1}(c_{i,l}/c_i)^{\varphi}$ , where  $\rho$  is a time preference parameter,  $c_i$  is consumption over period t,  $\varphi^{-1}$  is the intertemporal elasticity of substitution in consumption and  $\varphi$  the coefficient of relative risk aversion. Iterating forward the basic pricing equation  $E_{i,l}(R_i\zeta_i)=1$ , it is easy to see that if there are no bubbles  $P_{i,l}=E_{i,l}\left[\sum\limits_{k=0}^{\infty}\left(\prod\limits_{j=0}^{\infty}\zeta_{i+j}\right)D_{i+k}\right]$ , and hence  $P_{i,l}^*$  should be  $\sum\limits_{k=0}^{\infty}\left(\prod\limits_{j=0}^{\infty}\zeta_{i+j}\right)D_{i+k}$ 

Grossman and Shiller (1981) used aggregate consumption data to generate a serie for  $P_{t-1}^*$  for different values of  $\varphi$ . If there is risk neutrality, i. e.  $\varphi = 0$ , then  $\zeta_t = \rho^{-1} \ \forall t$ , and we go back to the constant expected return case an Shiller's (1981) results <sup>4</sup>. However, when the degree of relative risk aversion was 4, they could account for movements in stock prices from 1890 to 1950 but form 1960 onwards the results were not so encouraging. Unfortunately they did not do any formal testing so Kleidon's (1986a) criticism still applies. Besides, when using aggregate consumption data, all the usual criticism of the Consumption CAPM apply. But the point about time-varying expected returns is certainly crucial for all this literature. For example, Buiter (1987) proves that if the utility function is logarithmic (i.e.  $\varphi = 1$ ) and  $D_t$  is stationary, the inequalities will be reversed even asymptotically (see also Michener (1982)).

The unit root criticism gave rise to a series of second generation tests. Unfortunately, they still maintained the assumption of constant expected returns. One of the most ingenious is due to Mankiw, Romer and Shapiro (1985). They define a naive forecast of the stock price,  $P_{t-1}^0$ , as  $\sum_{k=0}^{\infty} v^{-(k+1)} E_{t-1}^0$  ( $D_{t+k}$ ), where  $E_{t-1}^0$  ( $D_{t+k}$ ) is a naive forecast of future dividends. Since under market rationality  $P_{t-1}$  is the best (in the mean square error sense) forecast of  $P_{t-1}^*$ , then the following inequality must be satisfied:

$$E(P_{\iota 1}^* - P_{\iota 1}^0)^2 \ge E(P_{\iota 1}^* - P_{\iota 1})^2$$

They derive their naive forecast under the assumption of static expectations, i.e.  $E^0_{\nu 1}$   $(D_{\nu +k}) = D_{\nu 1}$ , so that  $P^0_{\nu 1}$  will be  $(\nu -1)^{-1}D_{\nu 1}$ , i.e. the capitalized value of current dividends 5. Their results, though, imply that  $P^0_{\nu 1}$  was a better forecast of  $P^*_{\nu 1}$  than the actual share price.

An alternative volatility test which is also robust to the presence of unit roots was proposed by West (1988a). His test is based on the simple pre-

<sup>&</sup>lt;sup>4</sup> It is often believed that risk neutrality is also a necessary condition for constant expected returns. However, they could also be achieved with constant relative risk aversion if dividend growth rates are serially independent (see Ohlson (1977)).

<sup>&</sup>lt;sup>5</sup> As we shall see, this turns out to be equivalent to a stochastic detrending procedure which gets rid of the unit root.

mise that the (unconditional) mean square error of conditional expectation forecast errors is a non-increasing function of the amount of information included in the conditioning set. This implies in particular that

$$E[(P_t + D_t) - E(P_t + D_t | I_{t-1})]^2 \le E[(P_t + D_t) - E(P_t + D_t | H_{t-1})]^2$$

with  $H_{\iota_1} \subseteq I_{\iota_1}$ . He uses a restricted information set  $H_{\iota_1}$  which contains only lagged dividends. Assuming that  $D_{\iota}|H_{\iota_1}$  follows an ARI (p,d) process and that discount rates are constant, he finds evidence for excess volatility. Notice that both tests have a similar flavour. They both compare optimal forecasts with non-optimal ones. The non-optimality in Mankiw, Romer and Shapiro (1985) stems from the fact that static expectations are been used. By contrast, in West (1988a), conditional expectations are used, but they are based on a reduced information set.

The main lesson from the variance bounds literature reviewed here is simple: actual prices seem too volatile to be consistent with the simple present value formula with constant expected returns.

#### 3.2. Bubbles' Tests

If we were to put aside the theoretical restrictions discussed in section 2, bubbles could be a potential candidate to rationalize the excess volatility findings. If the bubble is uncorrelated with fundamentals, stock prices will be excessive volatile, whereas if the bubble is positively correlated with fundamentals, the market will overreact to news about dividends (see DeBondt and Thaler (1985, 1987)).

Despite the attractiveness of the bubble explanation, a word of caution is in order. If in order to compute  $P_{t-1}^*$  we set  $P_T^* = P_T$  (as in Mankiw, Romer and Shapiro (1985)), then we are implicitly allowing for bubbles under the null, and hence such volatility tests have no power to detect bubbles. Although other tests do not exactly use this condition, they will be partially affected too.

Therefore, bubble-specific tests have been devised. For example, West (1987) proposed a Hausman (1978) specification-type test for bubbles. His procedure can be illustrated with the following simple case. Assume that the restricted information set,  $H_{\nu 1}$ , contains only past dividend values, and that  $E(D_{\iota}|H_{\iota 1}) = \gamma D_{\iota 1}^{6}$ . Then, if we project the share price on  $H_{\iota 1}$ , we have that  $E(P_{\iota 1}|H_{\iota 1}) = \gamma (\nu - \gamma)^{-1} D_{\iota 1} + E(B_{\iota 1}|H_{\iota 1})$  since  $E(D_{\iota + \iota}|H_{\iota 1}) = \gamma^{\iota + 1} D_{\iota 1}$ . If there are no bubbles, OLS applied to this equation will give a consistent and efficient estimate of  $\gamma (\nu - \gamma)^{-1}$ , whereas it will be affected by an omitted variable bias under the alternative provided that the bubble is correlated with the dividend process. However, we can get a estimate of this coefficient which is consistent both under the null of no bubbles and under the alternative by combining the estimate of  $\gamma$  obtained from the univariate AR(1) model for  $D_{\iota}$  with an estimate of  $\nu$  obtained by using IV on the

<sup>&</sup>lt;sup>6</sup> This is quite different from assuming that  $E_{\iota\iota}(D_{\iota})=E(D_{\iota}|I_{\iota\iota})=D_{\iota\iota}$ .

equilibrium condition equation  $P_{t-1} = \nu^{-1} E_{t-1}(P_t + D_t)$ . The divergence between both estimates is measured in the metric of the Hausman test. West (1987) rejected the null hypothesis for US stock market data.

The problem of any such bubble test is that any unnoticed structural break in the dividend process can produce some misleading evidence for bubbles (see Flood and Hodrick (1986)). As an example, suppose that  $D_t$  is white noise with mean  $\phi$  until period T+1 when its mean is shifted by a constant  $\theta$ . The true  $P_T$  will be  $\phi$   $(\nu-1)^{-1} + \theta(\nu-1)^{-1}$ , which, by backward recursion, implies that  $P_T = \phi(\nu-1)^{-1} + \theta(\nu-1)^{-1} \nu^{-(T-1)}$ . But if we erroneously believe that  $D_t$  is a white noise process with mean  $\phi$  for ever, then we will expect  $P_{t+1}$  to be  $\phi(\nu-1)^{-1}$  and we would be «seeing» a deterministic bubble where there is none. Although this is an extreme example, Flood and Hodrick (1986) show that any bubble is observationally equivalent in finite samples to some switching in the dividend process.

Alternative bubbles tests are the unit root and co-integration ones used by Campbell and Shiller (1987). In order to carry out these tests, they define a new variable  $SL_{\mu 1}$ , as the difference between the current share price and the capitalized value of current dividends, i.e.  $SL_{\mu 1} = P_{\mu 1} - (\nu - 1)^{-1} D_{\mu 1} = P_{\mu 1} - P_{\mu 1}^{0}$  in the Mankiw, Romer and Shapiro (1985) notation. It is the easy to prove that if we call  $SL_{\mu 1}^* = (\nu - 1)^{-1} \sum_{k=0}^{\infty} \nu^{-k} \Delta D_{t+k}$  then  $SL_{\mu 1} = E_{\mu 1}(SL_{\mu 1}^*) + B_{\mu 1}$ . This means that if there are no bubbles,  $SL_{\mu 1}$  is the optimal forecast of a weighted average of future changes in dividends with geometrically declining weights. Hence if there are no bubbles,  $SL_{\mu 1}$  is stationary under the assumption that the dividend process has an arithmetic unit root. Besides, from the equilibrium condition it is clear that  $SL_{\mu 1} = E_{\mu 1} (\Delta P_t + \Delta D_t)$  so that  $P_{\mu 1}$  will have only one unit root under the no bubble, hypothesis. As a consequence  $P_{\mu 1}$  and  $P_{\mu 1}$  would be cointegrated with cointegration coefficient  $(\nu - 1)^{-1}$ . On the other hand; if there is a bubble,  $SL_{\mu 1}$  will not be stationary, neither will  $\Delta P_t$  and hence  $P_t$  and  $D_t$  will no longer be cointegrated of order (1,1).

Campbell and Shiller (1987) results appear to be mixed. Both  $P_{\nu 1}$  and  $D_{\nu 1}$  are I(1), but whereas evidence for cointegration is obtained when  $\nu$  is estimated, this is no longer the case when it is fixed to the sample mean (gross) return.

The problem with the cointegration tests is that many other models will also imply cointegration between prices and dividends. For example, the myopia model of Nickell and Wadhwani (1987), which assumes that  $P_{\iota_1} = E_{\iota_1}(\alpha P_{\iota} + \beta D_{\iota})$  with  $\beta > \alpha$ , also implies that  $P_{\iota_1} - \beta(1-\alpha)^{-1}D_{\iota_1}$  is stationary.

Because of the theoretical and empirical considerations that we have discussed so far, nowadays rational bubbles do not seem a very plausible explanation for the movements in stock prices. Besides, since they maintain the assumption of informational efficiency, they are unable to explain the empirical evidence on return predictability that we discuss next.

## 3.3. Cross-equation Restrictions Tests

Before the volatility tests were proposed, the most common tests of the efficient markets model with constant expected returns were based on autoregressions that tried to predict stock returns using their lagged values. In contrast with the variance bound results, these tests usually found almost negligible autocorrelations. This fact was initially interpreted as evidence in favour of the basic model (but see the discussion in section 4). The rational expectations literature suggested yet a third way to test the present value relation via the implied cross-equation restrictions on vector autoregressions. The Campbell and Shiller (1987) framework can be used to compare the single equation predictability tests and the cross-equation restrictions tests on VARs.

As we have seen, under the maintained assumption of no bubbles and constant discount rates, a unit root in  $D_t$  implies a unit root in  $P_{t-1}$  but cointegration between both variables. If we work in levels the unit root problem hampers inference. If in first differences, not only we lose information about level relationships, but also we end up with a non-invertible representation of the process (see Engle and Granger (1987)). A useful representation of the model is in terms of  $D_t$  (or  $P_t$ ) and  $SL_{t-1}$ .

Assuming that the infinite moving average (Wold) representation of the two variables is invertible and can be closely approximated by a VAR equation of order p, the whole system can be written as an expanded VAR(1) model in the variable  $z_t = (\Delta D_t,..., \Delta D_{t-p+1}, SL_t,..., SL_{t-p+1})$ . For the case of p = 2, we get:

$$\begin{bmatrix} \Delta D_t \\ \Delta D_{\iota_1} \\ \vdots \\ SL_t \\ SL_{\iota_1} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \\ 1 & 0 & 0 & 0 \\ \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta D_{\iota_1} \\ \Delta D_{\iota_2} \\ \vdots \\ SL_{\iota_1} \\ SL_{\iota_2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ 0 \\ \vdots \\ u_{2t} \\ 0 \end{bmatrix}$$

or

$$z_t = A z_{t-1} + v_t$$

Let  $H_{\iota_1}$  be the information set generated by  $\{z_{\iota_1}, z_{\iota_2}, ...\} = \{D_{\iota_1}, P_{\iota_1}, D_{\iota_2}, P_{\iota_2}, ...\}$  and  $SL_{\iota_1}^* = E(SL_{\iota_1}^*|H_{\iota_1})$  the unrestricted VAR forecast of  $SL_{\iota_1}^*$ . Defining the selection vectors  $g' = (0 \ 0 \ | \ 1 \ 0)$  and  $h' = (1 \ 0 \ | \ 0 \ 0)$ , and using the VAR(1) prediction formula we obtain  $E(\Delta D_{\iota_1\iota_1}|H_{\iota_1}) = h'A^{\iota_1}z_{\iota_1}$ . Hence  $SL_{\iota_1}^* = E(SL_{\iota_1}^*|H_{\iota_1}) = (\nu-1)^{-1}h'A(I-\nu^{-1}A)^{-1}z_{\iota_1}$ . Since  $SL_{\iota_1}$  belongs to the  $H_{\iota_1}$  information set,  $SL_{\iota_1} = SL_{\iota_1}^*$  if the model is correctly specified. That is, it must be the case that  $g'z_{\iota_1} = (\nu-1)^{-1}h'A(I-\nu^{-1}A)^{-1}z_{\iota_1}$ , which imposes the non-linear restrictions  $g' = (\nu-1)^{-1}h'A(I-\nu^{-1}A)^{-1}$ .

These are the usual cross-equation restrictions in models with rational expectations (cf. Hansen and Sargent (1981)). Despite being highly non-linear, they can be certainly tested. But if we post-multiply both sides by

 $(I-\nu^{-1}A)^{-1}$ , we end up with  $g(I-\nu^{-1}A) = (\nu-1)^{-1}h^2A$ , which are now linear, and extremely easy to test.

Imposing these restrictions on the VAR coefficients, after some tedious algebra we are left with  $E(\xi_l|H_{\nu l})=0$ , where  $\xi_{\nu l}=(R_l-\nu)P_{\nu l}$ . Hence the cross equation restrictions are equivalent to a predictability test for returns using information in  $H_{\nu l}$  only. This interpretation has the advantage that it is robust to the misspecification of the VAR representation, so that e.g. specifying the «wrong» order will only result in a loss of power. Besides, if we increase the dimension of the VAR by including other variables apart from lagged prices and dividends in the restricted information set, the crossequation prediction tests will be equivalent to a predictability test on the larger information set (see e.g. Campbell and Shiller (1988b)).

Campbell and Shiller (1987) tests rejected the basic model. But the advantage of computing  $SL'_{i-1}$  is that they could see the size of the difference between this variable and  $SL_{i-1}$ , which is equal to  $E(\sum_{k=0}^{\infty} \nu^{-(k+1)} \xi_{i+k} | H_{i-1})$ . Under the null hypothesis this expression should be 0, and the difference found in practice should only be attributable to random fluctuations due to the estimation process. On the other hand, such a difference will give us a measure of overall predictability of returns.

## 3.4. Orthogonality Tests

But perhaps the most obvious approach to see whether  $E_{\iota l}(P_{\iota l}^*) = P_{\iota l}$  is simply to regress one on the other. Under the null the coefficient of  $P_{\iota l}$  in this regression should be 1, and the coefficients of all other variables in  $I_{\iota l}$  (including a constant) should be  $0^7$ . One of the most powerful tests devised, and one less known is that of Scott (1985). He regressed  $P_{\iota l}^*$  on  $P_{\iota l}$  (and a constant), tested whether the coefficient was unity, and rejected the null hypothesis.

Durlauf and Hall (1989) use this framework to express most previous tests and evaluate their power. As an example, consider the regression of  $P_{\iota_1}$  on  $P_{\iota_1}^* - P_{\iota_1}$ , and call  $\gamma$  the (theoretical) regression coefficient. Under the null of market efficiency  $\gamma$  should be 0. But from the definition of  $\gamma$ ,  $1+2\gamma=[V(P_{\iota_1}^*)-V(P_{\iota_1})]/V(P_{\iota_1}^*-P_{\iota_1})$ , so Shiller's (1981) inequality will be reversed if and only if  $\gamma > -1/2$ . Since this is a weaker restriction than the orthogonality condition  $\gamma=0$ , the variance bounds tests actually lack power. The intuition is that the variance bounds literature tests a derived implication from the orthogonality of  $u_{\iota_1}$  with respect to the information set, namely that the variance of the forecast should be smaller than the variance of the actual outcome. Not surprisingly, when Durlauf and Hall (1989) extend Scott's tests by including several other variables in the

<sup>&</sup>lt;sup>7</sup> A regression coefficient of unity implies that  $V(P_{\iota 1}) = \text{cov}(P_{\iota 1}, P_{\iota 1}^*)$ . Hence prices should vary only in as much as they forecast future dividends (see Cochrane (1991)).

projection set like lagged prices and dividends, they find overwhelming rejections of the model.

Why then return autoregressions tend to find almost negligible forecastability of one period ahead returns whereas the projection and volatility tests systematically reject? To answer this question it is necessary first to understand what  $u_{\nu 1} = P_{\nu 1}^* - P_{\nu 1}$  is. Since under the null  $\xi_i = (P_t + D_t) - E_{\nu 1}(P_t + D_t)$ , the basic equilibrium relation  $P_{\nu 1} = \nu^{-1} E_{\nu 1}(P_t + D_t)$  can be rewritten in terms of  $\xi_i$  as  $P_{\nu 1} = \nu^{-1} (D_t + \xi_i) + \nu^{-1} P_t$ . Solving this equation forward and ruling out bubbles we obtain  $P_{\nu 1}^* - P_{\nu 1} = u_{\nu 1} = \sum_{k=0}^{\infty} \nu^{-(k+1)} \xi_{t+k}$ . Thus, a regression of  $P_{\nu 1}^* - P_{\nu 1}$  on lagged variables is essentially a regression of a geometrically weighted average of all future returns in excess of their constant expected value. As we shall see in the next section, if stock prices contain a slow-moving persistent component, this sort of long-run regressions can be more powerful than a single-period return autoregression.

## 4. Fads Models and Time-Varying Returns

Given the systematic rejections of the present value model with constant expected returns, it perhaps worth exploring alternative theories of share price determination. One such theory was proposed by Shiller (1984) in the context of a rational expectations framework. In his model there are two kind of agents. The first group, «smart money», are assumed to have a demand function for shares of the form  $Q_{-1} = [E_{-1}(R_l) - \rho]/\mu$ , where  $Q_{-1}$  is the fraction of shares that they hold,  $\rho$  is the return at which the demand for shares by this group is zero and  $\mu$  is the risk premium needed to induce them to hold all the shares. On the other hand there are also «ordinary investors» whose demand for shares,  $\Upsilon_{-1}$ , is exogenous. One example could be positive feedback traders, (i.e. agents who buy when stock prices rise and sell when they fall) in which case  $\Upsilon_{-1} = \gamma(R_{-1} - 1)$  with  $\gamma > 0$  (see Sentana and Wadhwani (1992)). Market equilibrium implies  $Q_{-1} + \Upsilon_{-1} = 1$ , from where

$$E_{\mu 1}(R_t) = \rho + \mu - \mu \gamma (R_{\mu 1} - 1)$$

Notice that if we did not have ordinary investors, we simply would have the equilibrium condition  $E_{\iota_1}(R_i) = \rho + \mu$ . More importantly, when  $\Upsilon_{\iota_1} \neq 0$ , if the smart investors are risk neutral, i.e.  $\mu = 0$ , an arbitrage argument implies that we are also back to the standard model. But if «smart investors» are risk averse, i. e.  $\mu \neq 0$ , fads imply predictability. The rationale is that the presence of ordinary investors increases uncertainty, and although there are certainly arbitrage opportunities for risk neutral individuals, risk averse ones are not willing to take them (see De Long, Shleifer, Summers and Waldman (1990)). As a consequence, the share price is different from the share price that would prevail if no ordinary investor existed. This difference is often call a fad.

Therefore, testing for predictability is an obvious way to test for fads. Summers (1986) showed, though, that testing for fads may be not so simple.

He assumes that the dividend process follows a geometric random walk, i.e.  $d_t = g^* + d_{+1} + w_t$ , with  $w_t \sim iid \mathcal{N}(0,\sigma^2)$ . Then it follows that the present discounted value of expected dividends is simply identical to the one implied by Gordon's growth model, i.e.  $P_{t-1} = E_{t-1}(D_t)/(v-g)$ , where  $g = \exp(g^* + 0.5\sigma^2)$ , which implies that the fundamental value is a geometric random walk itself. Suppose now that the actual log share price,  $p_t$ , is the fundamental value,  $p_{t-1}^*$ , plus a fad,  $a_t$ , where the fad is characterised by an AR(1) process of the form  $a_t = \emptyset a_{t-1} + v_t$ , and  $v_t \sim iid \mathcal{N}(0,\omega^2)$  independent of  $w_t^8$ . Then, the pseudo-returns process  $\Delta p_t$  will follow a negatively serially correlated ARMA(1,1) process, whose first autocorrelation coefficient is  $-(1-\emptyset)(1+\emptyset)^{-1}[2(1+\emptyset)^{-1} + \sigma^2/\omega^2]$ .

Summers (1986) point is that the serial correlation test will have extremely low power if  $\emptyset$  is close to one, i.e. if the fad component is very persistent. For instance, if  $\sigma^2 = \omega^2$  and  $\emptyset = 0.98$ , the first order autocorrelation will be approximately 0.005 and one would need 160,000 observations to be able to reject at the 5% level. The fad will introduce excess volatility because the variance of  $\Delta p_t$  will exceed that of  $u_t$  by  $2\omega^2/(1 + \emptyset)$ , but again it may be difficult to detect in practice.

The obvious answer to gain power it to decrease the data frequency. If we consider instead k-period «returns», i.e.  $\Delta_k p_t = p_t - p_{t,k}$ , the first autocorrelation coefficient takes the form  $\rho_1(k) = -(1-g^k)^2 (1+g^2)^{-1} [2(1-g^k) (1-g^2)^{-1} + k\sigma^2/\omega^2]$ . Fama and French (1988a) prove that for k small, the fad component dominates and there is an increase in negative serial correlation. However for k large, the random walk component of share prices dominates and the autocorrelations will go to 0.

This is closely related to the concept of mean reversion. The fact that  $a_t$  is mean reverting causes negative autocorrelation in returns. One way of testing for mean reversion is to use the variance ratio test. If  $p_t$  is a random walk with drift, then  $V(\Delta_k p_t) = kV(\Delta p_t)$ . The variance ratio test is defined as the ratio of these variances. For negatively correlated series, the variance ratio will be less that 1, whereas the opposite happens if there is positive serial correlation. Lo and Mackinlay (1988) use a Hausman-type specification test to compare  $V(\Delta p_t)$  with  $k^{-1}V(\Delta_k p_t)$  for different k-s. The first estimator should be more efficient since it uses all observations whereas the other one only uses 1/kth of them. They use weekly returns and reject the null hypothesis of geometric random walk. But they find positive rather than negative serial correlation. Although there are reasons to induce serial correlation in returns at high frequencies such as non-trading or bid-asks spreads, at the weekly level this is not completely convincing.

The variance ratio test can also be related to autocorrelations in multiperiod returns since  $VR(k) = 1+2 \sum_{i=1}^{k} (1-i/k)\rho_i(1)$ . Fama and French (1988a) prove that the first autocorrelation of  $\Delta_k p_t$ ,  $\rho_1(k) = VR(2k)/VR(k) - 1$ . They find

<sup>&</sup>lt;sup>8</sup> The independence of  $w_i$  and  $v_i$  is a convenient assumption. However, as Campbell (1991) shows, it is not inconsequential.

evidence for mean reversion using data for 1 to 10 year returns. In their results,  $\rho(k)$  reaches a maximum for 3 to 5 year returns, which corresponds to the period of highest predictability since the  $R^2$  of the regression of  $\Delta_k p_{t+k}$  on  $\Delta_k p_t$  is  $\rho(k)^2$ . Poterba and Summers (1986) also find similar results.

If we recall that the orthogonality and variance bounds tests can also be interpreted as multiperiod regressions, it now becomes clear in the light of the previous discussion why they could have more power than single period autoregressions against persistent fad-type deviations from the present value model with constant expected returns (see Campbell (1992)). The main difference is that they use geometrically weighted returns over the whole sample period instead of equally weighted returns over a fixed number of periods.

Other studies have found some predictability in returns using variables other than lagged returns, such as dividend yields (Fama and French (1988b)) or earning ratios (Campbell and Shiller (1988b)). They also find that, as in the mean reversion literature, the predictive power of these variables is very small for one-period returns but increases when one uses returns at lower frequencies. Notice that these results are consistent with the pre-volatility tests findings that one-period ahead returns were almost unforecastable.

However, multiperiod regressions have an overlapping dependent variable, which induces serial correlation in the residuals. This is a common problem in financial econometrics (see Hansen and Hodrick (1980)) and asymptotic corrections to the t-statistics (e.g. Newey and West (1987)) are nowadays a routine procedure that all previously mentioned papers took into account. But in finite samples such corrections may be misleading, especially if k is large relative to the sample size, since the standard asymptotic theory assumes that k/T goes to 0 when the sample size, T, increases. The intuition is that even when T is large (say a century), there are not very many nonoverlapping observations on k-period (say 5-year) returns. In this respect, Richardson and Stock (1989) propose the use of a Monte Carlo procedure to find asymptotic critical values which are valid when k grows proportionally with the sample size. They also show that such critical values provide a more reliable approximation in finite samples. A related procedure has been suggested by Kim, Nelson and Startz (1991), who compute finite sample significance levels by a bootstrap procedure in which they re-shuffle the original return data timewise. Both studies find that the evidence for mean reversion is weaker when one uses these alternative critical values.

An alternative way around the overlapping residuals problem is to regress single period returns on a (weighted) average of lagged values of the right hand side variables (see Cochrane (1991), Jegadeesh (1989) and Hodrick (1991)). The formal justification is that the moment condition  $E\left[\sum_{j=1}^{k} (R_{i+j} - \nu) \cdot x_{i+1}\right] = 0$  is identical to the condition  $E\left[(R_i - \nu) \cdot \sum_{j=1}^{k} x_{i+j}\right] = 0$  if  $R_i$  and  $x_i$  are jointly

stationary. The intuition is that if one-period returns contain a persistent predictable fad-type component, a persistent right hand side variable may help to pick it up.

Once more, the most serious problem from the economic point of view is that if expected returns are not constant, the fad can always be interpreted as the difference between the log share price and the log present discounted value of future dividends assuming constant returns. In fact Poterba and Summers (1988) show that if  $E_{l-1}(R_l) = \emptyset E_{l-2}(R_{l-1}) + \eta_l$ , then  $\Delta p_l$  will also follow an ARMA(1,1) process. In this respect, Campbell (1990) shows that a smooth process for expected returns can account for the behaviour of stock prices if it is persistent. The intuition is that a small but persistent change in next period expected return can have a large effect on today's stock price because the price depends on all future expected returns.

Similarly, the predictability of returns on the basis of lagged variables such as dividend yields or earning ratios may be due to the fact that these lagged variables are picking up the time variation in expected returns.

Ideally, we would like to combine an economically sensible model for timevarying expected returns with all the tests for constant expected returns discussed in section 3. Unfortunately, the present value formula on which most of them are based becomes highly non-linear when we allow for time-varying expected returns. An ingenious attempt to solve this problem is given by Campbell and Shiller (1988a) based on a log-linearized version of the model.

Let  $h_t \equiv \log(R_t) = \log(P_t + D_t) - \log(P_{t-1})$  denote the log gross return, and  $\delta_t$  the log dividend price ratio,  $d_t - p_t$ . Then, if we do a first-order Taylor expansion of  $\log(P_t + D_t)$  as  $\kappa + \rho p_t + (1 - \rho)d_t$ , it is easy to see that  $h_t$  can be approximated by  $\kappa + \delta_{t-1} - \rho \delta_t + \Delta d_t$ . The important result is that this approximation works quite well<sup>9</sup> in practice because dividend yields are typically small.

In order to transform this numerical approximation into an economic model of stock returns, Campbell and Shiller (1988a) make the assumption of rational expectations so that  $E_{i,1}(h_i) \simeq \kappa + \delta_{i,1} - \rho E_{i,1}(\delta_i) + E_{i,1}(\Delta d_i)$ , or

$$\delta_{\kappa_1} \simeq -\kappa + E_{\kappa_1}(h_t - \Delta d_t) + \rho E_{\kappa_1}(\delta_t)$$

Solving this equation forward, and assuming that «bubbles» are ruled out by the transversality condition, we obtain a present value formula for  $\delta_{L1}$ :

$$\delta_{t+1} \simeq E_{t+1} \left[ \sum_{k=0}^{\infty} \rho^k (h_{t+k} - \Delta d_{t+k}) \right] - \kappa (1 - \rho)^{-1}$$

This formula can be interpreted as saying that the current dividend yield is an increasing function of future discount rates and a decreasing function of future dividend growth.

<sup>&</sup>lt;sup>9</sup> In the case of  $d_i$  being a random walk, we have seen that  $P_i = gD_i/(\nu - g)$  so that they log-linearization becomes exact. For that reason Campbell and Shiller (1988a) call this model the dynamic Gordon model.

Once this formula is derived, the Campbell and Shiller (1987) methodology discussed above can be applied to this model with  $\delta_{\iota_1}$  replacing  $SL_{\iota_1}$  and  $h_t - \Delta d_t$  replacing  $\Delta D_t$ . In fact, although  $h_t$  and  $\Delta d_t$  enters symmetrically, one could actually separate the component attributable to  $\Delta d_t$  from  $h_t$  and gauge the contribution of each of these two components. In this respect, Campbell and Shiller (1988a) found that the contribution from  $\delta_t$  was more important that the contribution from  $h_t$ .

The main advantage, though, is that the model can be tested in conjunction with some explicit asset pricing theory for  $E_{\iota_1}(h_i) = E_{\iota_1}(\log R_i)$ , therefore relaxing the assumption of constant discount rates (i.e.  $E_{\iota_1}(\log R_i) = h$ ). For example, one can assume that there is a constant risk premium so that  $E_{\iota_1}(\log R_i) = c + \log R_{0\iota_1}$ , where  $R_{0\iota}$  is the (conditionally) safe interest rate. Alternatively, we could assume that  $E_{\iota_1}(\log R_i) = k + \varphi E_{\iota_1}(\Delta \log c_i)$  using a version of the consumption CAPM (see part II, section 4). Campbell and Shiller (1988a) found that the different models for expected returns did not seem to do a very good job.

#### 5. Conclusions

There is nowadays overwhelming evidence that the simple present value model for stock prices with constant expected returns is clearly rejected by the data. Surprisingly enough from a historical perspective of just over a decade, this fact is widely accepted by all parts to the debate: the believers in the efficiency of the stock market, The atheists and the agnostics.

The disagreement is now centred around whether the observed predictability of stock returns is due to rational variation in expected returns induced by shocks to preferences and/or the existence of time-varying risk, or it is simply the result of market inefficiencies. So far, attempts to explain such predictability in terms of current asset pricing models have not encountered much success, but we cannot jump to the conclusion that the stock market is irrational. Unfortunately, we may never see a test of market efficiency that is completely robust to the choice of asset pricing model. In any case, whether we can find an economically sensible asset pricing model that can account for the observed properties of stock prices is still an empirically meaningful question. As a first step, we shall discuss several empirically orientated asset pricing theories in the second part of this review.

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#### Resumen

Este trabajo presenta algunas de las contribuciones más significativas a la literatura empírica sobre la racionalidad del mercado de valores. El objetivo principal de esta literatura, que se ha concentrado en los aspectos dinámicos del mercado de valores en su conjunto, es contrastar si las rentabilidades descontadas son predecibles bajo una hipótesis mantenida sobre la tasa de descuento.

Hay hoy evidencia indiscutible que rechaza que el modelo del valor presente con rentabilidades esperadas constantes sea adecuado para los precios de las acciones. La controversia está ahora centrada en si la predecibilidad observada se debe a variación racional en las rentabilidades esperadas o es el resultado de la ineficiencia del mercado.

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