

BORROWING CONSTRAINTS IN ECONOMIES WITH INDIVISIBLE HOUSEHOLD CAPITAL AND BANKING: AN APPLICATION TO THE SPANISH HOUSING MARKET (1982-89)

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In this paper we quantify the individual, aggregate and welfare consequences of imposing different levels of borrowing constraints in a model economy where households make large, lumpy purchases of household capital which can be used as collateral for mortgages and consumer credit. We calibrate the model economy to 1982 Spanish data and we compare the steady state of a model economy where households can mortgage up to 50% of the value of their household capital with the one that obtains in a model economy where households can mortgage up to 80% of the value of their household capital. We ask whether our model economy can roughly account for the 120% increase in outstanding mortgages observed in the Spanish economy between 1982 and 1989 after a similar measure was adopted. We find that in our model economy some of the consequences of this policy change are the following: i.) aggregate loans increase by 147%, ii.) output increases by 2% and most of this increase is accounted for by the imputed rent of owner occupied housing, iii.) the stock of household capital increases by 12%, and iv.) household welfare increases by 0.6% of total household wealth. We conclude that borrowing constraints and indivisible household capital seem to play significant macroeconomic roles, and that this class of model economies seem to be a promising tool for the analysis of the housing market. (JEL G21, G28, E21)

1. Introduction

The passing of the Mortgage Market Regulatory Act on March 17, 1982 starts a period of significant changes in the Spanish housing market.¹

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¹See Levenfeld (1988) for a detailed report of these changes.

Some of these changes were the following: *i.*) the borrowing limit imposed on mortgages was raised from 50% to 80% of the value of the house being mortgaged, and *ii.*) in real terms the total value of outstanding mortgages rose from PTE 2,286 million in 1982 to PTE 5,030 million in 1989 —a 120% increase.²

The objective of this paper is double: first, we want to find out whether a change in the quantity of mortgages outstanding in the ball-park of the one observed in the 1982–89 period in Spain can be accounted for by relaxing the borrowing limit from 50% to 80% of the resale value of the household capital in a model economy calibrated to 1982 Spanish data, and, second, in so doing we want to provide a quantitative illustration of the role played by borrowing constraints in economies with indivisible household capital. More specifically, we want to quantify the individual, aggregate and welfare consequences of imposing different levels of borrowing constraints in a model economy where households make large, lumpy purchases of household capital which can be used as collateral for mortgages and consumer credit.

Much has been written about borrowing constraints. The empirical literature has found conclusive evidence that borrowing against future income is hard.³ The theoretical literature has explored the consequences of imposing borrowing constraints on model economy households subject to uninsured idiosyncratic risk from two different perspectives: the first approach followed by Schetchman (1976), Bewley (1977, 1980), and Deaton (1991), amongst others, explores the implications of borrowing constraints for individual savings decisions. The second approach, followed by Scheinkman and Weiss (1986), İmrohoroğlu (1989), Marcet and Singleton (1991), Díaz-Giménez (1997) and Díaz-Giménez and Prescott (1997), amongst others, focuses on the implications of borrowing constraints for the aggregate behavior of the economy.

The main differences between this paper and the previous literature on borrowing constraints are the following: First, the households in our model economy buy household capital which is indivisible and which costs several times the household's yearly income. This feature, which has been traditionally abstracted from in the literature, opens up a potentially important margin through which borrowing constraints

²These figures are taken from Freixas *et al.* (1991). The original data source is the Banco de Crédito Hipotecario, and they have been deflated using the Spanish Consumer Price Index.

³See Zeldes (1989) and the literature reviews of Hayashi (1987) and Deaton (1992).

affect the household decisions. Second, the households in our model economy can borrow using their household capital as collateral. This feature allows us to relax the extreme form of borrowing constraints that has been traditionally used in the literature. Third, the interaction between the indivisible capital and the borrowing constraints is particularly interesting, since it gives households an additional motive to save to make the required down-payments, and it also makes them balance the utility that they derive from owning their household capital against the cost that they incur when paying their mortgages and the maintenance costs. Finally, we think that modeling this form of collateralized borrowing against indivisible capital is potentially important because there is ample evidence that in real world economies this type of borrowing is large.⁴

Our model economy is very similar to the model economy described in Díaz-Giménez *et al.* (1992). A brief description of this economy is the following: It is inhabited by a continuum of households who live exponential life-times and are subject to an uninsured household-specific disturbance that affects their preferences, the market value of their time and their probability of dying. These households derive utility from consumption and from the services provided by a stock of indivisible capital which they can mortgage and which is costly to maintain and to sell. There is also a government sector and a banking sector. The government sector issues interest bearing debt and non-interest bearing reserves, it taxes labor and net interest income, it provides a public good, it makes transfers to indigent households, and it imposes a reserve requirement on banks. The banking sector intermediates between households by making collateralized loans to households who want to borrow and by accepting deposits from households who want to lend, and it also intermediates between the household and the government sectors, by pooling household savings and buying government-issued debt and reserves. We assume that banking intermediation is a costly activity and that these intermediation costs create a wedge between borrowing and lending rates. Finally, households make labor, consumption, investment, borrowing and lending decisions, and they have three reasons to save: to make the down-payments on their household capital, to substitute for insurance against the household-specific shocks, and to finance their consumption when they move into retirement.

⁴For instance, in 1986 in Spain mortgages represented almost 50% of total household borrowing, while in that same year in the U.S. this number was close to 75%.

Unlike Díaz-Giménez *et al.* (1992) who use U.S. data to calibrate their model economy, we calibrate our model economy to 1982 Spanish data. Once the model economy has been calibrated we carry out two computational experiments. In Experiment 1 households can mortgage up to 50% of the value of their household capital. In Experiment 2 we increase this borrowing limit to 80%. The purposes of this exercise are to replicate some of the changes introduced in the Spanish Housing Market after the passing of the Mortgage Market Regulatory Act on March 17, 1982, and in so doing to explore whether this class of model worlds can be used for the quantitative analysis of the housing market.

It goes without saying that any model economy must, per force, abstract from many features that are potentially important for the issues being considered. Technical considerations have lead us to abstract from many features of the housing market in this analysis. Specifically, the production side of our model economy is trivial since we assume that output can be costlessly transformed into consumption, household capital and the rental services of household capital. These limitations should be kept in mind when appraising our findings.

Some of these findings are the following: *i.*) aggregate loans increase by 147%, *ii.*) output increases by 2% and most of this increase is accounted for by the imputed rent of owner occupied housing, *iii.*) the stock of household capital increases by 12%, and *iv.*) household welfare increases by 0.6% of total household wealth. We conclude that indivisible household capital and financial arrangements seem to play significant macroeconomic roles, and that this class of model economies seem to be a promising tool for the analysis of the housing market.

The rest of the paper is organized as follows: In Section 2 we describe the model economy and we define its equilibrium. In Section 3 we give a detailed discussion of our calibration choices. In Section 4 we describe the policy experiment and we report our findings. Finally, in Section 5 we offer some concluding comments.

2. The model economy

The model economy analyzed in this paper is a version of Díaz-Giménez *et al.* (1992). This model economy includes three sectors: the government sector, the banking sector and the household sector. We describe the model economy population dynamics and each of these sectors in the subsections below.

2.1 Population dynamics

We assume that the model economy is inhabited by a measure one continuum of households. Every period, each one of these households faces an idiosyncratic random disturbance, s_t . This disturbance affects the household production opportunities, its utility function and its probability of dying. We assume that these disturbances are identically and independently distributed across households and that they follow a finite state Markov chain with transition probabilities given by

$$\pi_s(s' | s) = Pr\{s_{t+1} = s' | s_t = s\}, \quad [1]$$

where $s, s' \in S = \{1, 2, \dots, n_s\}$. We also assume that the Markov chain that generates s is such that it has a single ergodic set, no transient states and no cyclically moving subsets.

The state n_s is an absorbing state and it corresponds to death. Function $\sigma(s_t)$ is used to indicate whether or not a household is alive. If $\sigma(s_t) = 1$ we say that the household is alive at period t and if $\sigma(s_t) = 0$ we say that it is dead. When a household dies it does so overnight and it is replaced by a new household. Therefore, each period, the measure of new households is equal to the measure of households that did not survive from the previous period, and, consequently, the size of the population remains constant.

2.2 The government sector

The government in this economy performs the following functions: i.) it taxes labor and net interest income and estates, ii.) it transfers income to households, iii.) it provides a public good, iv.) it requires banks to keep a fraction of their deposits in reserves and, v.) it issues two assets.

The labor and net interest income tax rate is denoted by θ . It is assumed to be a proportional tax and it is restricted to being constant. The government also taxes households' estates. When a household dies, its estate is liquidated, the proceeds are used to pay off its debts and the remaining assets, if any, are subject to a 100 percent estate tax. The transfer policy is denoted by $\omega(a, k, s)$. It is contingent on the household's beginning-of-period real financial assets, a , on its tangible assets, k , and on its idiosyncratic shock, s .

One of the assets issued by the government bears no interest and it determines the unit of account. We denote it by R , and we call it

reserves. The other asset is a risk-free promise to deliver one unit of reserves at the beginning of the period immediately after its date of issue. This asset sells at a discount. We denote it by B , and we call it a T-bill.⁵

Variable p_t is the price of one unit of the date t composite good expressed in terms of reserves. We assume that government policy determines both the pricing process on p_t , $\varepsilon = p_{t+1}/p_t$, and the nominal interest rate on government debt, ι . We also assume that interest income is paid in advance.⁶ To implement these policies, the government exchanges goods and reserves at a price p_t and buys and sells promises to deliver one unit of reserves next period at price $1 - \iota$. We only consider economies with a positive nominal interest rate policy, that is, where $\iota \geq 0$. To induce a demand for the lower-yielding reserves, the government requires banks to keep at least a fraction ρ of their customers' deposits in reserves. Additional legal constraints preclude households from holding T-bills directly. Hence, only the financial intermediaries have access to the T-bill market.

A government policy arrangement is, therefore, a specification of $\{\theta, \omega(a, k, s), \varepsilon, \iota, \rho\}$, and the associated processes on public consumption, g , on the government supply of T-bills, B_g , and on the government supply of reserves, R_g .⁷

This specification of government policy can have two different interpretations. First we can think of it as an inflation and nominal interest rate targeting policy similar in essence to the policies followed by many governments and central banks in the real world. An implication of this type of targeting in the model economy is that the implied process on government consumption is determined residually. An alternative interpretation of the model economies' g is to consider it as the sum of government consumption and net exports in a small open economy whose government recurs to borrowing and lending abroad to finance its budget. A technical discussion of government policy and of the procedure used to compute the implied process on g can be found in the definition of equilibrium below.

⁵Unless otherwise indicated, we follow the convention that capital letters denote nominal quantities and lowercase letters denote the real values of the corresponding variables in terms of current-period prices.

⁶The reasons that justify this assumption are discussed below.

⁷Strictly speaking, both the regime of estate taxation and the set of legal constraints enforced are also part of the government policy.

Under this specification for the government policy, the nominal version of the government budget constraint is the following:

$$p_t g_t + R_{gt} + B_{gt} + p_t \Omega_t = p_t \mathcal{T}_t + R_{g,t+1} + B_{g,t+1}(1 - \iota), \quad [2]$$

where \mathcal{T}_t denotes the real value of aggregate tax receipts, and Ω_t denotes the real value of aggregate transfers.

2.3 The banking sector

We assume that banks play two major roles in our model economies. Their first role is to intermediate between households by making loans to households who want to borrow and by accepting deposits from households who want to lend. Their second role is to intermediate between the household and government sectors, by pooling household savings and buying government-issued T-Bills and reserves.

We assume that both the deposit and the lending technologies are freely accessible and that they display constant returns to scale. Consequently, in equilibrium bank profits are zero. To obtain a spread between the borrowing and lending rates, we assume that both the deposit and the lending technologies are costly. Banks incur a cost of η_D units of value for each unit of value that they accept in deposit. Similarly they incur a cost of η_L units of value for each unit of value that they lend to the household sector. Finally, in order to render the bank's decision problem static, we assume that interest income on loans, government debt and deposits is paid in advance.

Given these assumptions, the banks in the model economy solve the following static maximization problem:

$$\max_{B_b, R_b, L_b, D_b} B_b + R_b + L_b - D_b \quad [3]$$

subject to

$$B_b(1 - \iota) + L_b(1 - \iota_L) + R_b + \eta_D D_b + \eta_L L_b \leq D_b(1 - i_D) \quad [4]$$

$$R_b \geq \rho D_b \quad [5]$$

$$L_b, R_b, D_b \geq 0. \quad [6]$$

Where B_b denotes bank purchases of T-Bills, R_b denotes bank purchases of reserves, L_b denotes bank loans to households, D_b denotes household deposits accepted by the bank, and i , i_L and i_D denote,

respectively, the nominal interest rates on T-Bills, loans and deposits. The objective function [3] is the end-of-period net worth of the bank. Inequality [4] is the cash-flow constraint and inequality [5] is the reserve requirement.

Given that we only consider policies with a positive nominal interest rate on T-bills, these assets always dominate reserves in rate of return and, at an optimum, inequality [5] holds with equality. For maxima to exist with strictly positive D_b and L_b , the interest rates must satisfy the following conditions:⁸

$$i_L = i + \eta_L \quad [7]$$

$$i_D = (1 - \rho)i - \eta_D \quad [8]$$

To prevent banks from going bankrupt, we assume that they only make loans that are fully collateralized. Specifically, households at most can borrow up to the resale value of their end-of-period capital.⁹

2.4 The household sector

Preferences

We assume that households are only concerned with their future consumption and leisure if they are alive. Consequently, they order their random streams of these goods according to

$$E \sum_{t=0}^{\infty} \beta^t \sigma(s_t) \{u_1(c_t, k'_t, \tau - n_t, s_t) + u_2(g_t)\} \quad [9]$$

where u_1 and u_2 are continuous and strictly concave utility functions; $0 < \beta < 1$ is the time-discount factor; $c_t \geq 0$ is household consumption; k'_t represents the services of the capital goods and consumer durables held by the household during period t ;¹⁰ τ is a household's endowment of productive time; n_t is labor; and g_t is public per-capita consumption.¹¹ Hence, $\tau - n_t$ is time allocated by the household to non-market activities, which we call leisure.

⁸Note equations [7] and [8] imply that the difference between the household borrowing and lending rates is the following: $(1 - \rho)i_L - i_D = (1 - \rho)\eta_L + \eta_D$.

⁹To explore the implications of imposing different borrowing constraints on the household, in some of the computational experiments whose results are reported in this paper the absolute value of this bound is reduced.

¹⁰In the rest of the paper we will refer to k' indistinctively as "household capital", "houses" or "homes".

¹¹Note that the measure of households is one and, consequently, per capita and aggregate public consumption coincide.

Productive opportunities

The household's date t production of the composite good is

$$w(s_t)n_t, \quad [10]$$

where $w(s)$ is that household's technology parameter. We assume that this composite good can be transformed into consumption, investment, banking services, or capital maintenance services on a one-to-one basis. When households choose to work, they are paid their marginal product. Therefore, $w(s)$ also equals the household's real wage. Following Rogerson (1988) and Hansen (1985), we assume a labor indivisibility. Hours of labor services provided, n_t , are constrained to belonging to the set $\{0, 1\}$. Where 1 corresponds to being employed and 0 corresponds to not being employed.

Household assets

We assume that household savings take the form of either household capital or bank deposits. Households can buy discrete amounts of household capital, $k \in \mathcal{K} = \{0, \kappa, 2\kappa, \dots, n_k\kappa\}$, where κ is a real number which is large in terms of household average annual income. We also assume that household capital has to be maintained. Each period capital owners incur a cost of $\mu > 0$ units of that period's composite good per unit of value of the capital used during that period. Finally, we assume that there is an irreversibility in the capital accumulation process. When a household decides to sell part of its capital stock, $k > 0$ units of capital are transformed into ϕk units of the composite good, where $0 < \phi < 1$.

As far as household deposits are concerned, we assume that they belong to the finite set, \mathcal{D} .

2.5 Household liabilities and borrowing constraints

Households can borrow from banks. We assume that bank loans are constrained to belonging to a finite set \mathcal{L} . Specifically, we assume that $0 \leq \ell \leq \phi k_{t+1} p_{t+1}$, where parameter ℓ denotes the maximum amount that households can borrow from banks, and we explore the individual, aggregate, welfare and distributional consequences of considering different values of ℓ .¹²

¹²Note that the upper bound of the constraint $0 \leq \ell < \phi k_{t+1} p_{t+1}$ formalizes the requirement that household loans are fully backed by the resale value of the capital

Initial endowment and liquidation of assets

Households are born with no initial endowment of assets. When their time comes to die, they do so overnight, after the current-period labor, consumption, investment, and savings have taken place. Early in the following period, the deceased household estates are liquidated. Their capital goods are transformed into units of that period composite good and they are sold in the market. The proceeds of this sale are used to pay off the household's loans, if any. Whatever is left over is subject to a confiscatory estate tax.

The households' decision problem

Let D_t denote the nominal household deposits; L_t the nominal household debt; A_t the beginning-of-period nominal asset holdings; and x_t^d and x_t^s , respectively, the current-period purchases and sales of investment goods. Then the nominal version of the household competitive decision problem is

Objective function

$$\max E \sum_{t=0}^{\infty} \beta^t \sigma(s_t) u_1(c_t, k'_t, \tau - n_t, s_t) \quad [11]$$

subject to the following constraints, one for each $t = 0, 1, 2, \dots$:

Budget constraint

$$\begin{aligned} p_t c_t + p_t x_t^d + p_t \mu k'_t + T(s_t) + i_L L_t &\leq A_t + p_t w(s_t) n_t + p_t x_t^s + L_t \\ + i_D D_t + p_t \omega(a_t, k_t, s_t), & \end{aligned} \quad [12]$$

where

$$T(s_t) = \theta [p_t w(s_t) n_t + i_D D_t - i_L L_t]. \quad [13]$$

Borrowing constraint

$$L_t \leq \ell \leq \phi k_{t+1} p_{t+1} \quad [14]$$

Law of motion of financial assets

$$A_{t+1} = D_t - L_t. \quad [15]$$

owned by the household. Recall that this constraint guarantees that banks never go bankrupt.

Law of motion of capital

$$k_t = k'_t = k_t + x_t^d - x_t^s / \phi. \tag{16}$$

The maximization is also subject to next period's real financial assets, A_{t+1}/p_{t+1} , belonging to a finite set \mathcal{A} ; labor services, n_t belonging to the set $\{0, 1\}$; and end-of-period capital assets, k'_{t+1} , belonging to the finite set \mathcal{K} . Finally, variables A_0 and k_0 are taken as given.

Let $d = D_t/p_t$ denote the end-of-period household real deposits, $l = L_t/p_t$ the end-of-period household real loans, and $a = A_t/p_t$ the beginning-of-period household real assets, all three valued in terms of the current period's composite good, and let $e = p_{t+1}/p_t$. Then the dynamic program solved by an (a, k, s) -type household is the following:

$$v(a, k, s) = \tag{17}$$

$$\max_{c, n, d, l, x^d, x^s, a', k'} \left\{ \sigma(s)u_1(c, k', \tau - n, s) + \beta \sum_{s'} v(a', k', s')\pi(s' | s) \right\}$$

subject to

$$c + x^d + d + \mu k' \leq a + (1 - \theta)[w(s)n + \iota_D d - \iota_L l] + x^s + l + \omega(a, k, s) \tag{18}$$

$$l \leq \ell \leq \phi k' e \tag{19}$$

$$a' \leq (d - l)/e \tag{20}$$

$$k' = k + x^d - x^s / \phi \tag{21}$$

$a' \in \mathcal{A}$, $k' \in \mathcal{K}$, $n \in \{0, 1\}$ and a, k given. Since the household's problem is a finite-state, discounted dynamic program, an optimal stationary Markov plan always exists.

2.6 Definition of equilibrium

In the goods and securities markets, the government is not small, and, therefore, we do not treat it as a price-taking agent. Instead, part of the specification of the economy is the policy arrangement employed. Our explicit policy arrangement includes the following features: a description of the markets that operate and of the rules that govern banks; the price p_t at which the government exchanges goods for deposits and the law of motion of these prices, $p_{t+1} = p_t \varepsilon$; the price $1 - \iota$ at which the government sells T-bills to banks; the reserve requirement, ρ ; the

income tax rate, θ ; and the transfer policy, $\omega(a, k, s)$. For such a policy arrangement, a definition of a stationary competitive equilibrium is the following:

The state of a household is the triple (a, k, s) . The measure of households of type (a, k, s) is $y(a, k, s)$. We let y denote the corresponding measure. The economy-wide state is the measure of households y .

A stationary equilibrium for a policy arrangement $\{\varepsilon, \iota, \rho, \theta, \omega(a, k, s)\}$, consists of six basic parts: a government policy $\{g(y), b_g(y), r_g(y)\}$, a household policy $\{c(a, k, s), d(a, k, s), l(a, k, s), n(a, k, s), x^d(a, k, s), x^s(a, k, s), a'(a, k, s), k'(a, k, s)\}$, a banking policy $\{b_b(y), l_b(y), r_b(y), d_b(y)\}$, interest rates $\{i, i_D, i_L\}$, an inflation rate e , a law of motion for the measure of household types $y'_{a', k', s'} = f_{a', k', s'}(y)$ and an invariant measure of household types \bar{y} , such that

i.) Given i, i_L, i_D , and ρ , the banking policy solves the banking maximization program described in equations (3)–(6) above.

ii.) Given i_D, i_L, e , and θ , the household policy solves the household's optimization program described in equations (17)–(21) above.

iii.) The goods market clears:

$$\sum_{a, k, s} y(a, k, s)[c(a, k, s) + x^d(a, k, s) + d(a, k, s)\eta_D + l(a, k, s)\eta_L + \mu k'(a, k, s)] + g(y) = \sum_{a, k, s} y(a, k, s)[n(a, k, s)w(s) + x^s(a, k, s)] \quad [22]$$

iv.) The asset markets clear:

$$b_b(y) = b_g(y) \quad [23]$$

$$l_b(y) = \sum_{a, k, s} y(a, k, s)l(a, k, s) \quad [24]$$

$$r_b(y) = r_g(y) \quad [25]$$

$$d_b(y) = \sum_{a, k, s} y(a, k, s)d(a, k, s). \quad [26]$$

v.) Household and aggregate behavior are consistent:

$$f_{a', k', s'}(y) = \sum_{a, k, s \in \Phi(a', k')} y(a, k, s)\pi(s' | s) + \psi_{a', k', s'}(a, k, s), \quad [27]$$

where

$$\begin{aligned} \Phi(a', k') = \\ \{(a, k, s) : a' = [d(a, k, s) - l(a, k, s)]/e, k' = k(a, k, s) + \\ x^d(a, k, s) - x^s(a, k, s)/\phi\} \end{aligned} \quad [28]$$

and where ψ specifies the measure of types for the newborn. In our world, all the mass of ψ is on (a', k') pairs for which $a' = k' = 0$, and the total measure of those who are born is equal to the measure of those who die.

vi.) The behavior of endogenous variables is consistent with the policy arrangement. For our class of policy arrangements, this requires that $e = \varepsilon$, $i = \iota$, and $g(y) \geq 0$.

vii.) Measure \bar{y} is invariant:

$$y' = y = \bar{y} \quad [29]$$

For the set of policy arrangements that we consider, there is at most one equilibrium. The computational procedure that we use to find the equilibrium is first to solve the household problem, which is a finite-state discounted dynamic program, and then use (22) to determine $g(y)$. If $g(y)$ is positive we have found the unique equilibrium given the policy arrangement. Otherwise, we have established that no equilibrium exists for that policy arrangement. Appendix 2 contains a description of the algorithm used to compute this equilibrium.

3. Calibration

Given that in our model economy there is no aggregate uncertainty we can show that its equilibrium path converges to a unique steady state with an invariant distribution of households indexed by the triple (a, k, s) . Consequently, steady-state interest rates, aggregate stocks and aggregate flows are constant. We calibrate the model economy of Experiment 1 so that selected steady-state statistics are close to the corresponding 1982 data for the Spanish economy. We report most of our parameter choices in Tables 1 and 2.

3.1 Model period

We want to match some of the model statistics to both quarterly and yearly data. For this reason we select our model period so that a quarter

of a year is an integer multiple of the model period. Because wages are paid more frequently we would like to have a shorter model period, but since computational costs increase with the number of model periods per year, we settle for one-eighth of a year as our model period.

3.2 *Population dynamics*

To match the values of steady-state stocks, we consider two working-age states: $s = 1$ and $s = 2$, and a retirement state $s = 3$. During their working age, households can either be in state $s = 1$, which corresponds to periods in which their labor market productivity is high, or in state $s = 2$, which corresponds to periods in which their labor market productivity is low. We interpret state $s = 1$ to be a period in which the household finds a job that matches its qualifications, and state $s = 2$ to be a minimum wage opportunity. We choose the productivity in state 2, $w(2)$, to be 28 percent of the productivity in state 1. This choice roughly matches the ratio between the average earnings in manufacturing and the minimum wage in Spain in 1982.¹³ Given that we normalize the yearly wages of the highly productive types to be 1, our choices for the productivity parameters are $w(1) = 0.125$ and $w(2) = 0.0265$.

The working life of a household is geometrically distributed, and its expected duration is chosen to be 33 years. The retirement life of a household is also geometrically distributed, and its expected duration is chosen to be 10 years. Finally, recall that state $s = 4$ corresponds to death, and that to keep total population constant we assume that each period, the measure of deceased households is equal to the measure of the newly-born.

We select the transition probabilities between states 1 and 2 to be such that the expected duration of state $s = 2$ is 15 months,¹⁴ and such that, each period of time, 85 percent of the workers have the high productivity parameter and the remaining 15 percent have the

¹³The source for the minimum wage is the *Anuario de Estadísticas Laborales* (Labor Statistics Annual) and the source for the average wage in manufacturing is the Instituto Nacional de Estadística (National Statistics Institute).

¹⁴We use the average duration of unemployment in Spain to proxy for this variable. We choose 15 months as a compromise value between the 20 months reported by Blanco (1995) from his study of the "Encuesta de condiciones de vida trabajo" (Living and labor conditions survey) and the 8–11 months reported by García Serrano (1995) from his study of the "Encuesta de la población activa" (Labor force survey).

low productivity parameter.¹⁵ To be consistent with these choices we assume that 85 percent of the newly-born are of type $s = 1$, and the remaining 15 percent are of type $s = 2$. Table 1 reports the transition probability parameters that satisfy these properties.

TABLE 1
Transition probabilities for the household-specific shock, s

From this period state, s		To next period state, s'			
		1	2	3	4
Working age					
High productivity	1	0.9783	0.0179	0.0038	0.0000
Low productivity	2	0.0996	0.8966	0.0038	0.0000
Retired	3	0.0000	0.0000	0.9869	0.0131
Dead	4	0.0000	0.0000	0.0000	1.0000

3.3 The government sector

In our model economy government policy specifies the nominal interest rate on government debt, ι , the inflation rate, ε , the average income tax rate, θ , the transfer policy to indigent retirees, $\omega(0, 0, 3)$, and the reserve ratio, ρ . Our choices for these parameters are the following: For the nominal interest rate we choose, $\iota = 15.5\%$.¹⁶ For the inflation rate we choose $\varepsilon = 12\%$.¹⁷ For the average income tax rate we choose, $\theta = 0.20$.¹⁸ For the transfers to indigent retirees we choose $\omega(0, 0, 3) = 0.02$ which is $1/6$ of the average income of workers.¹⁹ And, finally, for the reserve ratio we choose $\rho = 6\%$.²⁰

¹⁵To obtain this number we divide the average employment rate by the average participation rate reported by the Encuesta de la Población Activa in 1982.

¹⁶This choice approximately corresponds to the average nominal interest rate on short term Spanish government debt during 1982 (for the first semester we consider the yield on two-year government debt and for the second semester the yield on one-year government debt (Pagarés del Tesoro). These last titles were issued for the first time on June, 1982.

¹⁷This choice approximately corresponds to the annual rate of growth for the Spanish GDP Deflator in 1982.

¹⁸Monés, Salas and Ventura (1992) report that the effective average income tax rate in 1982 in Spain was approximately 20%.

¹⁹This choice approximately corresponds to the ratio between the average earnings in manufacturing and the average retirement pension in 1982 in Spain.

²⁰This value corresponds to the legally required reserve ratio of the Spanish economy in 1982.

3.4 *The banking sector*

We choose the nominal annual interest rate on deposits to be 8 percent and the nominal annual interest rate on loans to be 17.5 percent.²¹ These interest rates, along with the policy parameters, imply the η_D and η_L parameter values reported in Table 2 below.

TABLE 2
The calibrated model economy parameters¹

Parameter	Notation	Value
<i>Government Policy</i>		
Reserve requirement	ρ	0.06
Tax rate on labor and interest income	θ	0.20
Nominal interest rate on T-bills	ι	0.019 ²
Inflation rate process	ε	1.015 ³
Transfers to indigent retirees	$\omega(0,0,3)$	0.02
Transfers to others	$\omega(a,k,s)$	0.00
<i>Per unit banking costs</i>		
Deposits	η_d	0.00821
Loans	η_l	0.00250
<i>Household preferences</i>		
Private consumption share	α	0.333
Capital services share	α_k	0.108
Risk aversion	ψ	4.0
Time discount factor	β	0.9994
Public consumption constant	δ_g	0.0083
Productive time endowment	τ	2.22
Retirees' constant	δ_r	0.45
<i>Technology</i>		
Real value of the units of household capital	κ	3.00
Maintenance cost	η	0.00625 ⁴
Rental service coefficient	γ	0.027
Disinvestment cost	$1-\phi$	0.10
Real wage in state $s=1$	$\omega(1)$	0.1250
Real wage in state $s=2$	$\omega(2)$	0.0265
Probability of a newly-born being of type $s=1$	Γ_1	0.85
Probability of a newly-born being of type $s=1$	Γ_2	0.15

1 Per model period unless otherwise indicated

2 This value corresponds to approximately 15.5% p a

3 This value corresponds to approximately 12% p.a.

4 This value corresponds to approximately 5% p a.

²¹ These values approximately correspond to the average weighted interest rates paid and charged by the Spanish banks and savings and loans institutions in 1982 according to Cuenca (1994).

3.5 The household sector

We choose parameters $n_k = 1$ and $\kappa = 3$ and, consequently, the set of possible housing stocks is $\mathcal{K} = \{0, 3\}$. A household with $k = 3$ corresponds to a family owning a house with a value three times its annual income if it is highly productive and it chooses to work every model period.²² In our class of model worlds this parameter is potentially important since it determines the size of the down-payments and the cost of house financing.²³

For workers ($s = 1$ or $s = 2$) with $k = 3$, the utility function of private consumption is

$$U_1(c, 3, n, s) = [c^{\alpha-\alpha_k} k^{\alpha_k} (\tau - n)^{1-\alpha}]^{1-\psi} / (1 - \psi). \quad [30]$$

We select $\tau = 2.22$ so that $n = 1$ corresponds to people choosing to work 45 hours of the 100 weekly hours of productive time.²⁴

We choose $\alpha = 0.333$ so that this 45 hour workweek is close to the one that would have been optimally chosen by the households.²⁵ We choose $\alpha_k = 0.108$ so that $k = 3$ would be optimal if the household rented housing services at a rate equal to the sum of the real after-tax interest rate on borrowing and the maintenance cost. This maintenance cost μ is set to 0.00625 per period, which corresponds to 5% per year. Finally we assume that households incur a disinvestment cost of $1 - \phi = 10\%$ when they sell their houses.²⁶

Workers ($s = 1$ or $s = 2$) who have no capital ($k = 0$) can transform the composite good at a rate $1/\gamma$ into housing services. Their indirect utility function of private consumption is

$$U_1(c, 0, n, s) = \max[c_1^{\alpha-\alpha_k} c_2^{\alpha_k} (\tau - n)^{1-\alpha}]^{1-\psi} / (1 - \psi) \quad [31]$$

²²To choose the value of household capital we follow Naredo (1993) who reports a value of 3.06 for the stock of household tangible assets to GDP ratio for the Spanish economy in 1984. We have not been able to find reliable data for 1982 for this parameter.

²³To analyze the sensitivity of our results to changes in the value of this parameter, we solved a model economy in which the set of possible housing stocks was $\mathcal{K} = \{0, 1, 2, 3\}$, and we carried out the same policy experiment that we describe below. Even though quantitatively some of the differences across both sets of experiments were large, this change did not affect any of the qualitative features of our findings.

²⁴Note that this value includes time spent commuting to and from work.

²⁵See Kydland and Prescott (1991).

²⁶These last two choices are the same as those reported in Díaz-Giménez *et al.* (1992).

subject to

$$c_1 + \gamma c_2 \leq c. \quad [32]$$

We select $\gamma = 0.027$ which is twice the sum of the real borrowing rate and the maintenance cost. This value of γ is chosen so that owning a house dominates renting housing services. Consequently, households purchase a house as soon as they have saved enough to make the required down-payment.

To keep our computational costs manageable we choose a very parsimonious modeling of retirement. Specifically we assume that retired households derive no utility from either owning or renting household capital and that the labor productivity of retirees is zero. Consequently, retired households always choose not to work, and retired homeowners sell their homes immediately upon retirement. Therefore their utility function of private consumption for the retirees is

$$U_1(c, k, n, 3) = \delta_r [c^\alpha]^{1-\psi} / (1 - \psi). \quad [33]$$

The larger the parameter δ_r , the more important is consumption during retirement relative to consumption during the working period of a household's life. Hence, the larger the value of δ_r , the larger is the equilibrium stock of savings for retirement. Parameter $\delta_r = 0.45$, is selected so that aggregate deposits at banks in the calibrated model economy of Experiment 1 are close to 1982 Spanish data.²⁷ To choose a value for the relative coefficient of risk aversion, ψ , we follow Auerbach and Kotlikoff (1987) and we pick $\psi = 4$.

All living households value public consumption by

$$U_2(g) = \delta_g g^{\alpha(1-\psi)} / (1 - \psi). \quad [34]$$

The value for $\delta_g = 0.0083$ is chosen so that half of the population prefers a higher tax rate, θ , and the higher associated level of government expenditures, g , and the other half of the population prefers a lower θ and the lower associated level of g .

3.6 Units and bounds

In addition to the parameters already discussed, in order for the program described in equations [17]–[21] to be well defined, we must choose the number of points in set \mathcal{A} . In the two experiments described

²⁷Using the methods described in Puch (1995) we obtain a value of 0.91 for the 1982 household deposits to GDP ratio in the Spanish economy.

below we choose the maximum value of the asset holdings to be 6. This value corresponds to six times the yearly income of a household who is always highly productive, and we find that it is never binding in equilibrium. The minimum value of the asset holdings is determined by the borrowing constraints, ℓ_i , and it takes a different value in each experiment. Finally, in both experiments we choose a grid size of 0.0027. This choice implies that the real value of the model economies' unit of account corresponds to approximately 0.35 percent of average per capita yearly income of the model economy. If we take the Spanish average per capita yearly real income in 1982 to have been PTE 1.5 million the smallest currency unit of the model economies would be worth, approximately, PTE 5,000. We consider that this unit is sufficiently small for the purposes of this paper. Making this unit smaller raises computational costs significantly and has virtually no effect on our findings.

4. Findings

We carry out two computational experiments. The stochastic processes, the exogenous elements of government policy, the banking technology and the household problems are all identical across both experiments. The experiments differ in the borrowing constraints that limit household decisions and in the endogenous elements of government policy.

Recall that the banking technology specifies that loans have to be fully collateralized. Hence, households with no capital cannot borrow at all in either of the two experiments. For households that own capital the borrowing limits are the following: in Experiment 1 the households can borrow up to 50% of the value of their household capital. This was the legal borrowing limit in effect in Spain before the Mortgage Market Regulatory Act was passed on March 17, 1982. Consequently, in Experiment 1 the household borrowing constraint is, $l \leq \ell_2 = 1.50$. In Experiment 2 the borrowing constraint is relaxed and households are allowed to borrow up to 80% percent of the value of their household capital. This was the legal borrowing limit in effect in Spain after the Mortgage Market Regulatory Act was passed. Consequently, in Experiment 2 the household borrowing constraint is, $l \leq \ell_2 = 2.40$.

The purpose of these experiments is to quantify, the individual, aggregate, and welfare consequences of relaxing the borrowing constraints. In the subsections that follow we report our main findings.

4.1 *Individual effects*

In our model economies households are born with no assets. Each period, they observe their household-specific productivity shocks and they decide whether or not to work, and how much to consume, invest, deposit and borrow. When households do not own their home, their consumption bundles are made up of consumption goods and rented housing services, and because owning a house dominates renting it, they save to make the down-payments on their homes.²⁸ Once a household buys its home, if it is sufficiently lucky, it only sells it upon retirement. On the other hand, if it is sufficiently unlucky, and it cannot meet its interest and maintenance payments, the household sells its home and it awaits for better times to buy another one. Homeowners pay the capital maintenance costs and the interest on their loans if any, and they save for retirement. When a household moves into retirement, it sells its home immediately, and it uses its assets to finance their consumption. If a retired household runs out of financial assets before it dies, it lives off welfare. If a retired household owns some financial assets when it dies, its estate is taxed away by the government. In the subsections that follow we describe how the different levels of the borrowing constraints affect household decisions.

Labor decisions

The indivisibility in the provision of labor services and the share of leisure in the utility function are such that in both experiments when the market value their time is high, ($s = 1$), households choose to work and save until they reach the maximum level of total asset holdings, $a + k$. These maxima differ across the two experiments. In Experiment 1 the maximum is $a + k = 6.6853$, and in Experiment 2 it is $a + k = 6.6798$.²⁹

On the other hand, when the market value their time is low, ($s = 2$), households only choose to work when they have no other way to finance their current-period consumption. Specifically, in both model economies low productivity households only choose to work when their total asset holdings belong to the interval $(0.00, 0.0082)$.³⁰

²⁸Note that households also save to substitute for insurance against the shocks to their earning opportunities.

²⁹Note that these maxima are also the upper bounds of the ergodic sets of the corresponding model economies.

³⁰In both model economies low productivity homeowners never choose to work. Instead, they borrow against their homes to finance their consumption. When they

Investment decisions

All the investment in our model economies adopts the form of purchases of household capital, and the borrowing constraints have the most significant effects on these investment decisions.

In Experiment 1, households can only borrow up to 50% of the value of their household capital, and consequently they must save until their financial assets belong to the interval (1.4371, 1.448) before they can buy their homes.³¹ In Experiment 2 households can borrow up to 80% of the value of their household capital and hence they can afford to buy their homes when their financial assets belong to the interval (0.5416, 0.5825). In both cases, they buy their homes as soon as they have saved enough to make the down-payment and they subscribe a mortgage to finance the rest of the value.

Borrowing constraints also have significant effects on the disinvestment decisions of households. Given our calibration choices, in both model economies households only sell their homes when the market value of their time is low ($s = 2$) and their financial asset holdings are such that the borrowing constraints become binding. In Experiment 1, this happens when the household financial assets belong to the interval $(-1.4997, -1.4752)$, and in Experiment 2, when they belong to the interval $(-2.4, -2.3734)$.³² In both cases households choose to sell their homes because their labor income is not enough to meet their maintenance costs, their interest payments if any, and to finance their consumption.

4.2 Aggregate effects

In this section we discuss the aggregate changes of relaxing the borrowing constraints from 50% to 80% of the resale value of the household capital.

Aggregate flows

We report the normalized income and product accounts of the model

can no longer meet the interest payments on these loans they choose to sell their homes instead of working.

³¹The household decisions are not triggered by a single point because of the interaction of the labor services and asset holding indivisibilities.

³²Note that $a = -1.4997$ and $a = -2.4$ are the lower bounds of the ergodic sets of the model economies of Experiments 1 and 2, respectively. This is because when a household sells its home it no longer has any collateral against which to borrow.

economies in Table 3, and we discuss our main findings below.

Aggregate output: We find that aggregate output increases 1.8 percent when we relax the borrowing constraints. On the income side (see the second and third rows of Table 3), we find that this increase is accounted for by the increase in capital income which measures the imputed flow of services from owner occupied housing.³³ Labor income, on the other hand, is essentially constant. This result arises both from the indivisibility in the choice of labor services, and from our choices for the relevant preference and technology parameters. On the product side (see the eight bottom rows of Table 3), we find that most of the increase in output is accounted for by private consumption, even though in percentage terms the increase in investment (11.1) is significantly larger.

TABLE 3
The income and product accounts of the model economies ¹

	Exp 1	Exp 2	%Δ
Output	1.000	1 018	1.8
Income account			
Labor income	0 855	0.855	0 0
Capital income	0.145	0.163	12.4
Private Consumption	0.837	0.853	1.9
Goods	0.505	0.493	-2.4
Housing	0.271	0 304	12.2
Imputed rent	0.145	0 163	12.4
Maintenance	0.125	0.141	12.8
Banking services	0.061	0.056	-8.2
Investment	0.009	0.010	11.1
Public consumption	0.154	0.155	0.6

¹ Note that the values in this table have been normalized dividing through by steady state aggregate output of the model economy in Experiment 1, $y_1=0.7701$

Private consumption: We find that private consumption increases by 1.9 percent when we relax the borrowing constraints, and that the different components of consumption behave differently. Specifically, we find that the consumption of both goods and banking services decreases (-2.4 and -8.2 percent, respectively) and that the consumption of the two components of housing services, imputed rent and mainten-

³³ We define capital income as $Y_K = K'(r_l + \delta)$ where $r_l = i_l - (e - 1)$ is the real interest rate on loans and δ is the rate of depreciation which, given that the average household only sells its home when it retires, is defined to be $\delta = (1 - \phi)[1 - Pr(s' = 1, 2 | s = 1, 2)]$.

ce, increases (12.4 and 12.8 percent, respectively). Quantitatively, the increase in housing services more than compensates the reductions in the other two components of consumption.

Investment: We find that investment increases by 11.1 percent when we relax the borrowing constraints.³⁴ This result arises from the increased access to home ownership that results from the increased access to credit and from the disinvestment costs incurred by households when they sell their homes.³⁵

Public consumption: We report the government accounts of the model economies in Table 4. We find that when we relax the borrowing constraints public consumption and transfers increase, and that interest payments decrease significantly. These results arise from the fact that relaxing the borrowing constraints makes households transfer their wealth out of financial assets and into real assets. As a result of this portfolio effect both interest payments on government debt and seignorage revenues are reduced significantly.³⁶ Given that tax receipts remain virtually unchanged, the differences between these changes bring about the corresponding increase in public consumption. The level effects of all these changes are small.

TABLE 4
The government accounts of the model economies ¹

	Exp 1	Exp 2	%Δ
Governments receipts ¹			
Tax revenues	0.1709	0.1709	0.0
Government outlays			
Public consumption	0.1543	0.1553	0.6
Transfers	0.0266	0.0271	1.9
Interest payments	0.0976	0.0519	-46.8
Government deficit ²	0.1076	0.0634	-41.1

¹ Note that the values in this table have been normalized dividing through by steady state aggregate output of the model economy in Experiment 1, $y_1=0.7701$

² Note that the government deficit equals seignorage revenues

³⁴Note that in spite of this large percentage increase, the change in the level of investment is small.

³⁵Recall that we define investment as $x_t^d - x_t^s/\phi$ and that $0 < \phi < 1$. Hence, the disinvestment cost implies that measured investment increases with the number of houses that are bought and sold

³⁶Note that in this class of model economies the government deficit is financed with seignorage revenues.

Aggregate stocks

We report the normalized household balance sheets in Table 5, and we discuss our main findings below.

Aggregate physical capital: We find that the stock of capital increases by 12.0 percent as we relax the borrowing constraints. This is because as the down-payments are reduced, more households can afford to become homeowners. In the economy of Experiment 1, the value before normalization of the stock of capital is 1.93 ($= 2.51 \times 0.770$). This implies that approximately 64% ($= 1.93/3$) of the households are homeowners. On the other hand, in the economy of Experiment 2, the value before normalization of the stock of capital increases to 2.17 which implies that approximately 72% of the households own their homes.

TABLE 5
Household balance sheets ¹

	Exp 1	Exp 2	%Δ
Assets	3.40	3.55	4.4
Capital	2.51	2.81	12.0
Deposits	0.89	0.74	-16.9
Liabilities			
Loans	0.15	0.37	146.7
Net worth	3.25	3.18	-2.2

¹ Note that the values in this table have been normalized dividing through by steady state aggregate output of the model economy in Experiment 1, $y_1=0.7701$

Aggregate deposits: We find that aggregate deposits decrease by -16.9 percent as we relax the borrowing constraints. This result arises because households save less to make the down-payments on their houses, and that they transfer their financial assets into real assets.

Aggregate loans: We find that aggregate loans increase significantly when we relax the borrowing constraints. Specifically, when we change the borrowing limit from 50% to 80% of the value of the house, aggregate loans in the calibrated model economy increase 146.7%. In the 1982-88 period in the Spanish economy loans increased 120% after a similar regulatory change was adopted.³⁷ This result is not surprising. Our model economy is an early step in the study of the housing

³⁷Note that we have calibrated the model economy in Experiment 1 so that its steady state loans to output ratio (0.15) is close to the ratio observed in 1982 in Spain (0.10).

market, and as such it abstracts from many of its relevant features. Perhaps the most conspicuous of these abstractions is that we do not consider the prices of housing.³⁸ In Spain, during the 1982–88 period the average price of housing increased approximately 90%. This increase was almost 50% higher than the 61% increase experienced by the Spanish CPI during that same period. We conjecture that if we had included this change in the price of housing in our model economy, the increase in loans in the model economies would have been significantly smaller.

Aggregate net worth: Aggregate net worth decreases by –2.2 percent when we relax the borrowing constraints. Quantitatively, the increase in households loans more than compensates the increases in household deposits and capital holdings.

4.3 Welfare effects

In this subsection we describe a measure of the welfare benefits associated with switching from one policy to another. This measure coincides with the measure used in Díaz-Giménez *et al.* (1992) and is based on the compensation principle—that is, the sum across households of the amount that households must receive to be indifferent between the current policy with the compensation and the new policy without the compensation. In this section, we use π to denote the steady-state policy rules and borrowing constraints, *i.e.* $\pi = \{\varepsilon, \iota, \rho, \theta, \omega(a, k, s), \ell\}$. We define the total wealth W of a household to be the sum of the values of its financial assets, a , its tangible capital, k , and its human capital, h .

We define the human capital of a household, $h(s)$, to be the expected value of the household's labor endowment discounted at rate β .³⁹ Then the value of the stock of human capital of a household in state s is

$$h(s) = E \left\{ \sum_t \beta^t \sigma(s_t) w(s_t) \mid s_0 = s \right\} \quad [35]$$

and the value of the stock of wealth of an (a, k, s) household type is

$$W(a, k, s) = a + k + h(s). \quad [36]$$

³⁸This assumption is made for technical reasons.

³⁹We use discount factor β because our model world does not have a single market discount factor since borrowing and lending rates differ.

We say a household's wealth is scaled by factor $\lambda > 0$ if that household's initial assets, a_0 and k_0 , and all the market values of its time, $w(s_t)$, are scaled by factor λ .

Let $v_1(a, k, s \mid \pi)$ denote a household's value of its private consumption and leisure processes given that the current policy is π , and let $v_2(s, \bar{y} \mid \pi)$ denote a household's value of the public consumption process given that the current policy is π and the economy is in a steady state described by measure \bar{y} . Then the household's total value, which includes the value of both the private and the public processes is

$$V(a, k, s, \bar{y} \mid \pi) = v_1(a, k, s \mid \pi) + v_2(s, \bar{y} \mid \pi), \quad [37]$$

where v denotes the household's total value with the policy argument π made explicit.⁴⁰

In our model world, if the wealth of a household and its welfare payments, ω , are scaled by a factor λ , the total utility of that household is scaled by the factor $\lambda^{\alpha(1-\psi)}$. For the v_1 part of V , this follows immediately from the household's maximization problem described in (11)–(16).⁴¹ For the v_2 part of V , this result follows because the scaling results in the equilibrium $\{g_t\}$ process being scaled by λ given that the resource balance constraint (22) must hold. Let π_0 denote current policy, \bar{y} the current steady state distribution, and π_1 the alternative policy being evaluated. The compensation factor λ is

$$\lambda(\cdot \mid \pi_0, \pi_1) = [V(\cdot \mid \pi_1)/V(\cdot \mid \pi_0)]^{1/\alpha(1-\psi)}. \quad [38]$$

A household's benefits of switching from policy π_0 to π_1 is the product of its $(\lambda - 1)$ times its wealth W . If negative, we interpret these benefits as costs.

The welfare measure, M , is computed summing the benefits over all households alive at that point in time and dividing the resulting number by aggregate household wealth. Therefore:

$$M(y, \pi_0, \pi_1) = \frac{\sum_{a,k,s} W(a, k, s) [\lambda(a, k, s, y \mid \pi_0, \pi_1) - 1] y(a, k, s)}{\sum_{a,k,s} W(a, k, s) y(a, k, s)}. \quad [39]$$

⁴⁰Note that the expression for v_2 is:

$$v_2(s, \bar{y} \mid \pi) = E \left\{ \sum_t \beta^t \sigma(s_t) \delta_g g(\bar{y})^{\alpha(1-\psi)} / (1 - \psi) \mid \pi \right\}$$

⁴¹Note that the integer constraints must be modified from $k \in \mathcal{K}$ to $k/\lambda \in \mathcal{K}$ and from $a \in \mathcal{A}$ to $a/\lambda \in \mathcal{A}$.

This welfare measure can be interpreted of as corresponding to the benefit of a fractional wealth change that persists forever and that is of magnitude M .

Given the structure of our model economy, an issue is what is the share of the welfare costs associated with a policy switch that arises from the resulting changes in private consumption and what is the share that arises from changes in public consumption. To address this issue, we allocate a fraction $[v_1(\pi_1) - v_1(\pi_0)]/[V(\pi_1) - V(\pi_0)]$ of M to private consumption and a fraction $[v_2(\pi_1) - v_2(\pi_0)]/[V(\pi_1) - V(\pi_0)]$ of M to public consumption. Summing private consumption benefits over all households and dividing by total wealth yields a private benefit welfare measure M_1 . Following the analogous procedure for public consumption benefits yields welfare measure M_2 for public benefits. The sum of M_1 and M_2 is M .

TABLE 6
Welfare benefits that arise from relaxing the borrowing constraints (1)

Current policy	New policy	Benefits (% of W)		
		Total	Private	Public
$l=1.50$	$l=2.40$	0.60	0.55	0.05

We use this welfare measure to evaluate the cost of switching from the model economy described in Experiment 1 where households can borrow up to 50% of the value of their household capital and, hence, $l_1 = 1.50$, to the model economy described in Experiment 2 where households can borrow up to 80% of the value of their capital and, hence, $l_2 = 2.40$. We find that relaxing the borrowing constraints increases our measure of household welfare, but that the welfare increase was small: about 0.60% of aggregate household wealth. We also find that most of this increase, 0.55%, is accounted for by the change in private consumption, and that the change in public consumption only accounts for the remaining 0.05%. These findings are reported in Table 6.

5. Concluding comments

In Spain between 1982 and 1989 the value of outstanding mortgages increased by 120% in real terms. Amongst other changes, on March 17, 1982 a regulation was passed that raised the borrowing limit imposed on mortgages from 50% to 80% of the value of the house being

mortgaged. In this paper we ask whether a model economy in which borrowing constrained households make large and lumpy purchases of household capital which can be used as collateral for mortgages, can account for comparable changes. We find that in our model economy a similar policy change brings about an increase in aggregate loans of 147%, an increase in output of 2% and an increase in household welfare of 0.6% of total household wealth.

An obvious extension of this line of research is to model explicitly the production side of our economy. Ideally these extended economies would have a consumption sector, a housing sector, and a separate market for rental housing. Unfortunately, some of these extensions might prove to be beyond our computational capacity. Until this is done, our findings should be considered to be preliminary and tentative. Still, keeping these limitations in mind, our findings lead us to conclude that borrowing constraints and indivisible household capital seem to play significant macroeconomic roles, and that this class of model economies seem to be a promising tool for the analysis of the housing market.

Appendix: The computational algorithm

An outline of the algorithm used to compute the equilibria of the economies described in this paper is the following:

Step 1: Given the policy implied $\{e, i_D, i_L, \theta, \omega(a, k, s)\}$, solve the household decision problem described in [17]–[21] and obtain the vector of household decision rules $d(a, k, s)$.

Step 2: Given the initial distribution of household types, $y_0(a, k, s)$, and d , compute the following period distribution of household types, $y_1(a, k, s) = f(y_0, d)$.⁴²

Step 3: If $y_1(a, k, s) = y_0(a, k, s)$, we are done. Else update $y_0(a, k, s)$ and goto Step 2.⁴³

An outline of the algorithm used to solve the households' decision problem is the following:

Step 1.1: Impose a grid on the household state space $\{\mathcal{A} \times \mathcal{K} \times \mathcal{S}\}$.

⁴²Note that function $f(y_0, d)$ is the function defined in [27].

⁴³Note that throughout this computational procedure we must make sure that the implied process on public consumption, g , is non-negative. Otherwise we would have established that no equilibrium existed for the given government policy.

Step 1.2: Initialize the value function, $V_0(a, k, s)$, and the vector of decision rules, $d_0(a, k, s)$.

Step 1.3: Given V_0 , use Bellman's equation to compute the iterated value function, V_1 , and its corresponding iterated vector of decision rules, d_1 . This is achieved solving the following functional equation: $V_1(a, k, s) = \max_{d_1} \{\sigma(s)u_1(d_1, a, k, s) + \beta E[V_0(a', k', \{s'\})]\}$ subject to $(a', k') = g(d_1, a, k, s)$.

Step 1.4: If $V_1(a, k, s) = V_0(a, k, s)$, we are done. Else update $V_0(a, k, s)$ and goto Step 1.3.

As in most computational procedures that search over large state spaces, we use various techniques to improve the computational efficiency of this algorithm. These techniques are implemented in a documented FORTRAN program that is available from the authors upon request.

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Resumen

En este artículo se cuantifican las consecuencias de las restricciones de liquidez sobre las decisiones individuales, las variables agregadas y el bienestar, en una economía en la que los hogares compran viviendas que se puedan usar como colateral para obtener créditos hipotecarios. El modelo se calibra para la economía española con datos de 1982, y se compara el estado estacionario de una economía en la que los hogares pueden hipotecar el 50% del valor de sus viviendas con el que se obtiene cuando los hogares puedan hipotecar hasta el 80% del valor de sus viviendas. Este ejercicio nos permite evaluar la capacidad del modelo para justificar el incremento del 1205 en el valor de los préstamos hipotecarios observado en la economía española entre 1982 y 1989 tras la adopción de una medida similar. Los principales resultados de este ejercicio son los siguientes: i) los préstamos aumentan en un 147%, ii) la producción agregada aumenta en un 2%, iii) el valor del fondo de viviendas aumenta en un 12% y iv) el bienestar aumenta en un 0,6% de la riqueza total. Estos resultados nos permiten concluir que las restricciones de liquidez y las indivisibilidades asociadas a la acumulación del capital por parte de los hogares parecen jugar un papel macroeconómico importante y que los modelos de este tipo pueden ser un instrumento útil para el estudio de problemas relacionados con el mercado de la vivienda